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ON THE OPTICAL CONDITIONS REQUIRED TO SECURE MAXIMUM ACCURACY OF MEASUREMENT IN THE USE OF THE TELESCOPE AND SPECTROSCOPE.

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(*Continued from p. 299, Vol. 16, December 1902.*)

B. Causes of asymmetry in the image of Class B may be subdivided as follows:

B (1). Lack of resolving power due to (*a*) small aperture, (*b*) imperfect definition, or (*c*) imperfect achromatism.

B (2). Errors of adjustment of focus.

B (3). Errors due to an asymmetrical arrangement or use of the instrument itself.

B (4). Errors due to erroneous or imperfect design.

B (1). A frequent cause of error in the measurement of the position of a point or line source is an unsymmetrical broadening of the image by the superposition upon it of another image of a second fainter source too close to the former to be clearly "resolved."

If the two sources vibrate independently and have intensities

i_1, i_2, the distribution in intensity in the superposed images is represented by

$$i_1 \frac{\sin^2 \xi}{\xi^2} + i_2 \frac{\sin^2 (\xi - \kappa)}{(\xi - \kappa)^2}, \tag{54}$$

κ being the angular interval between the centers of the two sources, and ξ, the angular distance of any point in the image

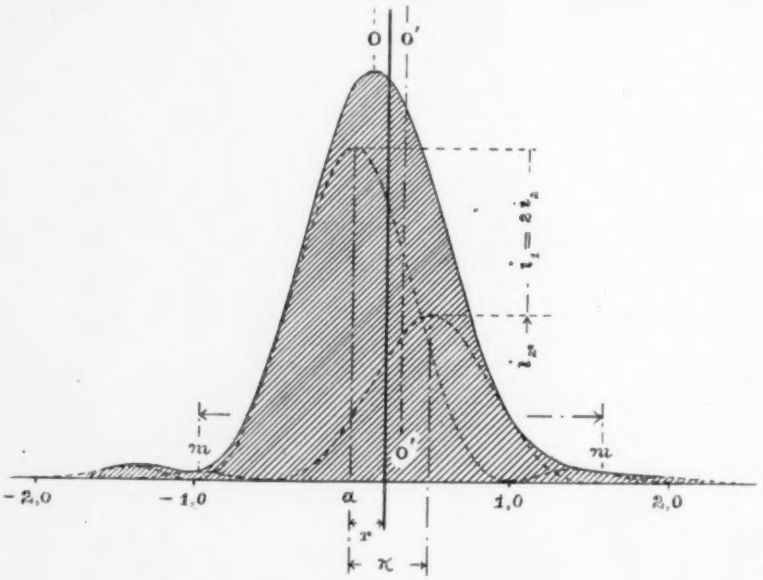


FIG. 9.

from the center of the geometrical image of the principal source S_1. An example of the image of such a double source is shown in Fig. 9. The dotted lines represent the separate diffraction images of the two sources, the full line the resulting focal image of the two together.

In measuring the position of such an image the reference cross-wires are always set too far to the right in the figure, *i. e.*, toward the smaller component, by an amount depending on its intensity in comparison with that of the principal source. Examples of errors of measurement of this kind are found in the determinations of position, proper motion, and parallax of very close double stars.[1] Similar errors in the determination of wave-

[1] Certain interesting cases of this kind which seem to have escaped attention will be considered fully in a subsequent paper. (Note added January 1903.) One of

lengths of spectral lines have been frequently noted.[1] The only way in which such errors can be detected and corrected is by the use of higher resolving power, by which the two components may be separated and measured separately. It appears probable from the work of Michelson[2] and the more recent work of Perot and Fabry,[3] that by the use of sufficiently high resolving powers a large number of the lines which are now regarded as single will be resolved into two or more components, in many cases of very unequal intensity.

In this case the error of setting the cross-wires is a real one, for the reason that we desire to determine in our measurements the position of one of the components of the double source (generally the brighter one) and not the mean position of the two. If we know the separation of the two sources and their relative magnitude, we may indeed correct the results of the measurements of the double source so as to give with a fair degree of approximation the true position of either component. In general the setting of the cross-wire on such an asymmetrical image as is here being considered will be a compromise between the tendency to set on the point of maximum intensity O, Fig. 9, of the compound diffraction image and the point O', midway between those points of equal intensity *mm*, which are just perceptible to the eye of the observer. In the case represented in Fig. 9, in which $\kappa = \frac{1}{2} a_0$ and $i_1 = 2 i_2$, the setting of the cross-wire would be about $r = a + 0.4\kappa$.

these cases was recently investigated by Comstock, whose paper, "The Motion of 85 *Pegasi*," was read on December 30, 1902, at the recent meeting of the Astronomical and Astrophysical Society in Washington.

[1] For example, see Rowland's "Table of Standard Wave-Lengths;" KAYSER and RUNGE, "Ueber die Spectren der Elemente," *Abh. d. K. Akad. d. W.*, Berlin, 1888–1892; HASSELBERG, "Untersuchungen u. d. Spectra der Metalle," *Köngl. Svenska Vet. Akad. Handlingar*, 1894–1898; KAYSER, "Influence of Slit-Width on Comet Spectra," *A. N.*, **3217**; *A. and A.-P.*, **13**, 367; HARTMANN, "Wave-Length of the Nebular Lines," ASTROPHYSICAL JOURNAL, **15**, 292; WRIGHT, *ibid.*, **16**, 57, etc.

[2] "Application of Interference Methods to Spectroscopic Measurements," *Phil. Mag.*, (5) **34**, 280.

[3] *Bull. Astron.*, **16**, 5; ASTROPHYSICAL JOURNAL, **9**, 87; *Jour. Phys.*, (3) **9**, 369; ASTROPHYSICAL JOURNAL, **15**, 73, etc.

The corrected reading for S_1 is, therefore,

$$a = r - 0.4\kappa \ , \tag{55}$$

and for S_2 similarly

$$a + \kappa = r + 0.6\kappa \ . \tag{56}$$

B (2). Displacements due to imperfect adjustments for focus.

In considering errors of measurement due to this cause alone we shall at first assume optical conditions to be such that the wave-front which forms the image is symmetrical and concentric about the optical axis on which the image is formed; *i. e.*, that there is no unsymmetrical aberration, either spherical or chromatic, and no eccentricity.

When these conditions are fulfilled a change of focus should result simply in a symmetrical expansion of the diffraction disk or band, which represents the physical image of a point or line.[1] The center of each image will remain fixed on the secondary optical axis joining the center of the geometrical image with the optical center of the lens system. Under such conditions the separation of any two such images at the focal distance z from the optical center of the objective is

$$\xi = 2\, z \sin \frac{\kappa}{2} \ , \tag{57}$$

where κ, as before, is the angular separation of the two images. Whence for any change in z

$$d\xi = 2 \sin \frac{\kappa}{2}\, dz = \kappa dz = dz \frac{\xi}{z} \ . \tag{58}$$

This expression is entirely independent of f, the true focal length of the instrument, and hence if z could always be maintained *constant*, dz and, therefore, $d\xi$, would be zero, no matter what the *absolute* value of z might be. This is only possible under absolutely constant temperature conditions. When the temperature changes, z will also change, first, on account of the thermal expansion or contraction of the telescope tube, and, second, on account of the mechanical refocusing which may be necessary in order to compensate for the corresponding tempera-

[1] This is a common way of testing the excellence either of an objective or of atmospheric conditions.

ture change in f. We can best consider these two effects separately.

1. The change in z, due to the thermal change in the length of the telescope tube, for a difference of temperature Δt will be

$$\frac{\delta}{dt} z = az\Delta t$$
$$= \frac{d\xi}{2 \sin \frac{\kappa}{2}} ,$$

where a is the coefficient of expansion of the material of which the telescope tube is made. But we have also a similar change in the scale of the measuring or recording device at the focal plane, which amounts to

$$- d\xi' = - a' \left(2z \sin \frac{\kappa}{2}\right) \Delta t .$$

The apparent change in the separation of the two images as indicated by the reading of the instrument is therefore

$$d\xi + (- d\xi') = 2z \sin \frac{\kappa}{2} (a - a') \Delta t = \Delta_m \xi . \qquad (59)$$

If the material of the micrometer screw or reference scale is the same as that of which the telescope tube is constructed, then $a' = a$ and $\Delta_m \xi = 0$; *i. e.*, in this case the measured separation of the images at the focal distance, z, remains the same at all temperatures.

If the position of the images is first recorded photographically on a sensitive plate, and afterward measured by a comparator or similar device, we have to consider not only the relative expansions of the glass plate and the telescope tube, but also those of the plate and the scale or screw of the comparator. In this case the total change in the measured distance as indicated by the reading of the comparator at a temperature $T = t_0 + \Delta' t$ is

$$\sum_{t}^{T} d\xi = 2z \sin \frac{\kappa}{2} \{(a - a') \Delta t + (a' - a'') \Delta' t\} . \qquad (60)$$

In order to make (60) vanish we must have

$$(a - a') \Delta t = (a'' - a') \Delta' t .$$

This condition may be satisfied in one of three ways: (*a*) by making $a'' = a$ and $\Delta' t = \Delta t$; *i. e.*, by making the screw or scale of the comparator of the same material as the telescope tube and measuring the plate at the same temperature at which it was photographed. This plan is generally neither convenient nor practicable; (*b*) by controlling the temperature, T, of measurement, so that for given values of a, a' and a'' we always fulfill the relation

$$T - t_0 = \frac{a - a'}{a' - a''} = (t - t_0) \ .$$

This is also inconvenient and troublesome, although less so than the preceding; (*c*) by making $a = a' = a''$; *i. e.*, by constructing both the telescope tube and the comparator scale of glass or platinum. This, while impracticable for large telescopes, is not at all so for small ones, such as are used for spectroscopes.

Instead of attempting to make the correction (60) vanish we may simply determine its amount and apply it to the measurements. In this case we need to determine the degree of accuracy with which the temperatures t_0, t, and T must be known. This is easily done by equating the differential of (60) with the limiting value of ϵ expressed in linear measure, *i. e.*,

$$\epsilon_0 = \epsilon f \cong \epsilon z \ .$$

This gives at once from (60) and (14)

$$\frac{\delta}{dt}\{(a - a')\Delta T + (a' - a'')\Delta' t\}\, dt = \frac{\lambda}{15 b\kappa} \ . \qquad (61)$$

The values of a, a', and a'' for brass, glass, and steel (the materials usually employed) are respectively

$$a \cong 0.000018 \qquad a' = 0.000008 \qquad a'' = 0.000010.$$

In case of the Mills spectrograph the aperture b is about 3.7cm. Hence for an angle $\kappa = 1^\circ\!.5 = 0.027$[1] we have for (61),

$$dt = \frac{40}{10\frac{\delta}{dt}\Delta t - 2\frac{\delta}{dt}\Delta' t} \ , \qquad (62)$$

and if we assume that it is equally easy to measure all three temperatures, t_0, t, and T, (62) indicates that the error (which

[1] See Table II of this paper.

must be taken without regard to sign), in determining any one of them must not exceed 1°.6. It is generally possible to be certain of all these temperatures to this or even a higher degree of accuracy, and it is therefore usually better to observe them and correct the observed readings of the comparator than it is to attempt to eliminate the corrections by any of the methods indicated in the preceding paragraph.

2. *Effect of refocusing:* In large telescopes the alteration of the true focal length, f, for a given change in temperature is so much greater than the corresponding thermal alteration in z alone, that it is necessary to refocus the eyepiece or plate in order to obtain good definition. Such refocusing is also essential, as will be shown later, even in small instruments, if there is any unsymmetrical aberration or eccentricity in the incident wave-front. In such cases we have

$$\begin{aligned} dz &= \frac{\delta}{dt}(z + d\Omega) + d\Omega \\ &= (z + \delta\Omega)\,a\Delta t + d\Omega \\ &= \frac{d\xi}{2\sin\frac{\kappa}{2}} \end{aligned} \tag{63}$$

where $d\Omega$ represents the actual mechanical movement of the eyepiece or plate in refocusing. The change in the separation of the two images at the new focal plane, as indicated by the micrometer screw or comparison scale, now is

$$\Delta\xi = 2\,(z + d\Omega)\sin\frac{\kappa}{2}\left\{(a - a')\,\Delta t + \frac{d\Omega}{z + d\Omega}\right\}, \tag{64}$$

and in the case of comparator readings on photographs

$$\Sigma\xi = 2\,(z + d\Omega)\sin\frac{\kappa}{2}\left\{(a - a')\,\Delta t + \frac{d\Omega}{z + d\Omega} + (a' - a'')\,\Delta' t\right\}. \tag{65}$$

If we assume that the terms of (64) and (65) which represent temperature corrections are computed as before and applied to the measurements, we have left to consider only the effect of the term

$$2d\Omega \cdot \sin\frac{\kappa}{2} = \kappa d\Omega\ .$$

In order that this may be computed also with the same degree of accuracy as already assumed we must have

$$d\,(d\Omega) \leqq \frac{\epsilon f}{\kappa} = \frac{\lambda}{30\beta\kappa}\ , \tag{66}$$

β being the semi angular aperture.

The degree of accuracy which it is necessary to observe in refocusing (*i.e.*, in determining the amount of mechanical shift of the eyepiece or plate with reference to the telescope tube) is therefore independent of the size of the instrument and depends only on its form (*i.e.*, its angular aperture), and the angular separation of the two objects measured. We will determine what this is in a few cases of particular interest.

For large visual telescopes and heliometers, $\beta \cong 0.027$. Hence for $\kappa = 0^\circ\!.35 = 0.0061$,[1] and $\lambda = 0.000056$ cm.,

$$d\,(d\Omega) \leqq 0.01 \text{ cm} \cong \frac{1}{250} \text{ in.} \tag{67}$$

For photographic telescopes and concave gratings of sufficiently long focus to give full photographic resolution $\beta \cong 1/80 \cong 0.013$. The field κ is, however, correspondingly larger. For Rowland's concave gratings, for example, $\kappa \cong 0.074$ (more than 4°). For such cases therefore

$$d\,(d\Omega) \leqq 0.002 \text{ cm}\ , \tag{68}$$

or less than one one-thousandth of an inch.

From the above results, (66), (67), and (68), it would appear necessary that the focusing scales of large telescopes designed for micrometric or heliometric work should be provided with verniers capable of reading to 0.1 mm, while those for the view telescopes of large spectrographs and spectrometers should be capable of reading to at least $1/50$ mm.

The accuracy of focusing demanded in all these cases is considerably higher than is usually considered necessary, or is attained. It is, of course, far in excess of either visual or photographic requirements on the standard of either resolution or general optical definition. But as we have already stated, this

[1] This is the limit imposed by considerations of aberrational distortion in the case of the Lick and Yerkes telescopes (see Table II). This limit has, however, been exceeded in a number of cases of individual measures with these instruments.

standard is an inadequate and unsafe one to apply as a test of the *accuracy* of an optical instrument.

In the case of a telescope used as a collimator the accuracy of focusing necessary depends on the nature and method of use of the instrument.

In general, any lack of exact collimation may be compensated by altering the focus of the observing telescope, and if the change in the latter is not objectionable and everything remains symmetrical with respect to the axis of collimation, no error can be thus introduced. In the case of prism spectroscopes, however, the condition of symmetry with respect to the x axis cannot be fulfilled, and any divergency or convergency in the beam of light coming from the collimator objective will result in the introduction of an unsymmetrical aberration in the wave-front traversing the prism train, the amount of which will depend on the curvature of the incident wave front, the optical perfection of the prisms themselves, and the angles of incidence on the successive faces of the prisms.

The simplest case to consider is that in which the axis of the incident wave-front traverses the train at minimum deviation, the faces of the prisms are optically plane, and the material of which they are composed is of uniform optical density. The amount of longitudinal aberration produced by the passage of a cone of light of semi-angular aperture θ through a single prism under the conditions assumed above has been calculated by Rayleigh, who finds[1]

$$\delta v = 3\theta u \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2 - 1)}{n^2}, \tag{69}$$

where i and i' are the angles of incidence on the first and second faces of the prism and u is the radius of the spherical wave-front.

We have also the relation[2]

$$\delta v = \frac{3E}{\beta^2}, \tag{70}$$

[1] RAYLEIGH, *Phil. Mag.*, (3) **9**, 40–49. See also "Theory of the Ocular Spectroscope," ASTROPHYSICAL JOURNAL, **16**, 2, July 1902.

[2] RAYLEIGH, *Phil. Mag.*, (3) **8**, 411. The result there given is for a symmetrical aberration depending on x^4. For an unsymmetrical aberration depending on x^3 the numerical factor in the numerator is 3, not 4.

where β is the semi-angular aperture of the cone of rays emerging at the second face of the prism. Hence if we denote the aperture of the collimator by b and the length of path of the central ray through the prism by L, we have

$$\theta_1 = \frac{b}{2v} \cong \frac{b}{2(u+nL)} \cong \theta\left(\frac{u}{u+nL}\right) . \qquad (71)$$

Also

$$\frac{1}{f} - \frac{1}{f+df} = -\frac{1}{u} , \qquad (72)$$

whence

$$u = -\frac{f(f+df)}{df} , \qquad (73)$$

and

$$\theta_1 = -\frac{bdf}{2\{f^2 + (f-nL)\,df\}} . \qquad (74)$$

In most cases the quantity $(f-nL)$, which represents the difference between the principal focal length of the collimator and the optical path of the central ray through the prism, is so small that the second term of the denominator in (74) is vanishingly small compared to f^2 and may be neglected.

We then obtain at once from (69), (70), (71), and (74)

$$E = \beta^3 \frac{(df)^2}{f} \cdot \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2-1)}{n^2} , \qquad (75)$$

where β as before denotes the semi-angular aperture of the collimator objective.

The limiting value of df will be found by equating (75) and (14), which gives

$$df = \pm \frac{n \cos i'}{4\beta} \sqrt{\frac{f\lambda}{\beta \tan i\,(n^2-1)}} . \qquad (76)$$

If we neglect the successive changes of the second order in the value of θ, which are indicated in (71), we have similarly for N prisms

$$E_N = NE ,$$

$$\therefore\ df_N = \frac{df}{\sqrt{N}} . \qquad (77)$$

In the case of a spectrometer which has a collimator of focal length $f = 100$cm and aperture $b = 5$ cm, and a prism train consisting of a single 60° prism of light flint of index 1.6 for $\lambda = 0.00005600$,

we have $\beta = 0.025$, $i = 53^\circ\ 7'\ 48''$, and $i' = 30^\circ$. For this case then

$$df = \pm 4.55 \text{ cm} , \tag{78}$$

which shows that under the conditions assumed above a very considerable range is permissible in focusing the collimators of even very large spectrometers.

This conclusion is so at variance with the commonly accepted statement that an accurate focusing of the collimator is very important[1] that it is liable to be misinterpreted unless carefully considered. It is only true when the minimum deviation condition is strictly fulfilled. It is necessary therefore to consider, first, the degree of accuracy with which the various parts of the spectrometer train may be adjusted and maintained in their correct relative positions, and second, the amount by which a given error in adjustment will affect the results given in (75) and (76).

With any given instrument it is always possible by simple hand adjustment alone to bring the prism to a position of minimum deviation with an error not exceeding that which would produce a displacement of the image equal to ϵ, the metrological power of the view telescope. The relation between the angular displacement θ of the prism from the position of minimum deviation and the resulting change in deviation of the central ray has been determined by the writer, who finds for small values of θ,[2]

$$\left.\begin{aligned} \Delta &= \Delta_0 + \sin^{-1}\theta^2\left(1+\frac{\theta^2}{2}\right)\frac{\sin\frac{\phi}{2}(n^2-1)}{n\cos^2\frac{\phi}{2}\sqrt{1-n^2\sin^2\frac{\phi}{2}}} \\ \text{or}\quad \delta &= \Delta - \Delta_0 \cong \theta^2\left[\frac{\sin i}{\cos i\cos^2 i'}\cdot\frac{(n^2-1)}{n^2}\right] \cong C\theta^2 \end{aligned}\right\} , \tag{79}$$

where Δ_0 is the angle of minimum deviation of the ray, and Δ

[1] See for example SCHUSTER, *Enc. Brit.*, article "Spectroscopy;" HARTMANN, ASTROPHYSICAL JOURNAL, **12**, 36; FROST, ASTROPHYSICAL JOURNAL, **15**, 15, etc.

[2] ASTROPHYSICAL JOURNAL, **2**, 280. The expression there given is in error. The term $\sqrt{1-n^2\sin^2\frac{\phi}{2}}$ should appear only in the denominator and not in the numerator.

the deviation when the prism has been displaced from the correct position for Δ_0 by an angle θ. If, therefore, we put $\delta = \epsilon \cong \frac{1}{16}\frac{\lambda}{b}$ we have for θ

$$\theta = \frac{1}{4} n \cos i \sqrt{\frac{\lambda}{b \tan i (n^2 - 1)}}, \tag{80}$$

an expression very similar to (76). For the case of the spectrometer above considered

$$\theta = 0.0008 \cong 3'.$$

This is about as accurately as the prism can be maintained at minimum deviation with the ordinary sliding link movement, unless great care is taken in its construction. With such a device therefore it is useless to set the prism with any greater exactness than is indicated above.

If, however, we use a pivoted link minimum deviation device or, better still, a fixed arm prism train,[1] we can control the movement of the prism much more accurately and can then use a more accurate method of initial setting, for example a theodolite, with advantage. We may thus with ease increase both the initial and continuous accuracy of adjustment at least three times, *i. e.*, we can reduce θ to not more than $1'$.

It is next necessary to determine the effect of a given value of θ on the aberration as given by (69) or (75).

The expression for the longitudinal aberration produced by the passage of a cone of light through a prism placed in any position with reference to the axis of the ray can be deduced from the general fundamental equations given by Rayleigh in the paper to which reference has already been made. The expressions thus obtained are, however, complicated and cumbersome to reduce, and for small angles of inclination to the position of minimum deviation the total aberration may be obtained in a more simple way in the following manner:

Let caa', coo'', Fig. 10, be the central and edge rays passing through the prism. Owing to the difference θ in the angles of incidence, the total deviation of the two rays in passing through the prism will differ from each other by a small angle $o'a'o'' = f(\theta)$

[1] *Phil. Mag.*, (5) **38**, 337, October 1894.

which we will call D. Since the wave-fronts are by supposition very nearly plane, the extreme edge aberration E between the incident wave-front ao and the refracted wave front $a'o''$ will be

$$E = o'o'' \cong ao \cdot D = u\theta D = u\theta f(\theta) \ . \tag{81}$$

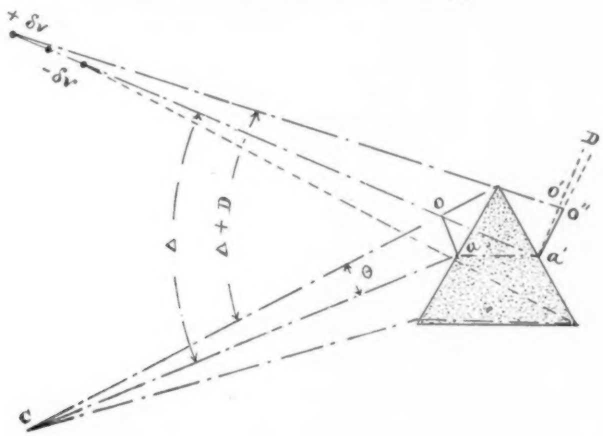


FIG. 10.

When the central ray falls upon the first face of the angle of incidence i for minimum deviation, we have from (79)

$$f(\theta) = \delta = C\theta^2 \ ,$$

and

$$E_o = Cu\theta^3 = \theta^3 u \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2 - 1)}{n^2} \ , \tag{82}$$

and from (70)

$$\delta v = 3\theta u \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2 - 1)}{n^2} \ , \tag{83}$$

the same expression as obtained by Lord Rayleigh.[1]

The method above indicated of determining the aberration of a prism applies not only to the position of minimum deviation but also to any other position within which equation (79) holds. When the central ray is incident on the prism at an angle

[1] It is perhaps worth while to note that the expression for the longitudinal aberration of a prism deduced by Abbot and Fowle, *Amer. Jour. Sci.*, (4) **2**, 253, and *Annals of the Smithsonian Astrophysical Observatory*, **1**, 78–79, is based upon an erroneous assumption. If equation (12) of their paper were correct there would be no aberration, *i. e.*, the angular deviation of the central and extreme rays would be the same.

other than that for minimum deviation, say at an angle $i \pm \kappa$

$$\Delta_c = \Delta_0 + f(\kappa)$$
$$= \Delta_0 + C\kappa^2 .$$

And for the extreme ray incident at an angle $i \pm (\kappa + \theta)$ we have similarly

$$\Delta_e = \Delta_0 + C(\kappa + \theta)^2 .$$

Hence

$$D = C(2\kappa\theta + \theta^2) , \tag{84}$$

and the wave-front aberration E is

$$E_\kappa = Cu\theta^3\left(1 + \frac{2\kappa}{\theta}\right) , \tag{85}$$

$$= \beta^3 \frac{(df)^2}{f^2} \cdot \frac{\tan i}{\cos^2 i'}\left(\frac{n^2 - 1}{n^2}\right)\left(1 - \frac{4\kappa f^2}{bdf}\right) , \tag{86}$$

As before we obtain the limiting values of df for a single prism by equating (86) and (14). This gives us after reduction

$$df = \frac{\kappa f}{\beta} \mp \frac{n \cos i}{4\beta}\sqrt{\frac{\lambda f}{\beta \tan i\,(n^2 - 1)} + \frac{16\kappa^2 f^2}{n^2 \cos^2 i'}} , \tag{87}$$

which for large values of κ may be reduced to the form

$$df = \mp \frac{1}{32}\frac{\lambda}{\kappa} \cdot \frac{n^2 \cos^2 i'}{\beta^2 \tan i\,(n^2 - 1)} . \tag{88}$$

When the prism train contains N prisms, each of which is out of the position of minimum deviation by the same amount κ, the total aberration E_N (to the same degree of approximation as before) is N times that for a single prism. If we call the first term under the radical in (87) A and the second term B, we have for N prisms

$$df_N = \frac{\kappa f}{\beta} \mp \frac{n \cos i}{4\beta}\sqrt{\frac{A}{N} + B} . \tag{89}$$

For $\kappa = 0$ we have as before

$$df_N = \frac{df}{\sqrt{N}} , \tag{90}$$

and for large values of κ

$$df_N = \frac{df}{N} , \tag{91}$$

i. e., the effect of adding to the number of prisms in the train is

greater as the departure from the position of minimum deviation increases.

The permissible values of df both for a single prism and for a train of three prisms have been computed for the spectrometer already considered (in which $f = 100$ cm, $\beta = 0.025$ and $\mu = 1.6$ for $\lambda = 5600$ tenth-meters) for a series of successively increasing values of κ from $\kappa = 1'$ to $\kappa = 10°$. They are given in columns 2 and 3 of Table III.

TABLE III.

κ	df_κ, $N=1$	df_κ, $N=3$	K	df_κ, $N=1$	df_κ, $N=3$
	cm	cm		cm	cm
0° 0′ 0″	−4.55	2.62	....	....	.....
0° 1′ 0″	−3.54	1.77	....	....	.....
0° 3′ 0″	−2.25	0.90	....	....	.....
0° 15′ 0″	−0.57	0.21	....	....	.....
0° 30′ 0″	−0.26	0.12	0° 30′ 0″	0.57	0.21
1° 0′ 0″	−0.15	0.06	1° 0′ 0″	0.26	0.12
1° 30′ 0″	−0.10	0.035	1° 30′ 0″	....	.....
2° 0′ 0″	−0.07	0.023	2° 0′ 0″	0.15	0.06
2° 15′ 0″	−0.066	0.022	....	....	.....
3° 0′ 0″	−0.049	0.016	3° 0′ 0″	0.10	0.035
....	....	.....	4° 0′ 0″	0.07	0.023
5° 0′ 0″	−0.029	0.010	....	....	.....
10° 0′ 0″	−0.016	0.005	10° 0′ 0″	0.03	0.010

As κ increases, the permissible values of df decrease, at first very rapidly and then much more slowly. For large values of κ df must be very small; *i. e.*, the focusing must be very exact, especially with several prisms. This is the reason why Schuster's method of focusing (in which the prism is turned far out of the position of minimum deviation, first on one side and then on the other) gives such accurate results. Thus Table III shows that when the displacement κ is 10°, errors of focusing of only 0.016 cm, or of 1 part in 6,000, become manifest with a single prism. It is probably for this reason also that the statement already alluded to on p. 11 has been so often made.

Further consideration of the values of df for such values of κ as may be likely to occur because of errors of adjustment, shows, however, that so far as this cause is concerned we still have a very considerable latitude in the adjustment of the collimator. Thus if the error of adjustment κ does not exceed $3'$, as assumed

on p. 12, the focusing of the 100 cm collimator need not be closer than 2 cm for one prism or 0.9 cm for three prisms. For $\kappa = 1'$, errors of focusing as large as 3.5 cm and 1.75 cm for one and three prisms respectively may be permitted.

In the case of the spectrograph, in which a considerable angular field is under simultaneous observation, it is of course impossible to adjust the prisms to minimum deviation for all parts of the field at once. If we suppose that the prisms are accurately adjusted to minimum deviation for the central image in the field (wave-length λ_0) and that the maximum semi-angular field, corresponding to spectral images of wave-lengths λ and λ', is K, the amount κ by which a single prism is out of minimum deviation for these wave-lengths is

$$\frac{1}{2}(K \pm \delta) = \frac{1}{2}\{K \pm c(\kappa^2)\}$$
$$\text{and} \quad \kappa \cong \frac{1}{2}K\,. \tag{92}$$

The required accuracy of focusing is obtained as before from (87). Values of df for $K = 30'$, $1°$, $2°$, $3°$, $4°$ and $10°$ ($\kappa = 15'$ to $5°$) are given in Table III, column 5.

When the prism train consists of N prisms and the maximum semi-angular field is K_N as before, the amount by which each prism of the train is out of minimum deviation is:

$$\text{for the first prism,} \quad \kappa_1 \cong \frac{1}{2N} K_N\,;$$
$$\text{for the second prism,} \quad \kappa_2 = \frac{3}{2N} K_N\,;$$
$$\text{for the third prism,} \quad \kappa_3 = \frac{5}{2N} K_N\,;$$
$$\text{and} \quad \text{for the } N\text{th prism,} \quad \kappa_N = \frac{2N-1}{2N} K_N\,.$$

The total aberration, E_N, in the train is, under such circumstances [see (85)],

$$E_N = Cu\theta^3 \sum_{\kappa_1}^{\kappa_N} \left(1 + \frac{2\kappa}{\theta}\right),$$
$$\cong C\beta^3 \frac{df^2}{f} \left\{ 1 - \frac{4f^2}{bdf} \cdot \frac{K_N}{2} \right\} N\,, \tag{93}$$

and the permissible errors df for the train are therefore the same as for a train of N prisms each of which has an error of adjustment κ equal to

$$\frac{K_N}{2}.$$

The value of df for this case for three prisms and for the same semi-angular fields as before are given in the last column of Table III.

It will be seen that even for semi-angular fields as large as 2° (total 4°) the permissible range in focus of a 1 meter collimator is nearly ¼ mm, a comparatively large quantity compared to that which expresses the permissible range in focus of the view telescope. The statement already made that the accuracy required in focusing the collimator is much less than that required in focusing the view telescope is therefore generally true, not only for single prism spectrometers, but for large multiple prism spectrographs with fields of 3° or 4°.

In this latter case, however, the range of focus is not sufficiently great to permit us to collimate over more than a limited range of wave-lengths or to disregard altogether changes in temperature in the apparatus. In a well-designed objective of the usual two-lens type, of the size and angular aperture assumed above, the change in focus, df_λ, is between $0.0004f$ and $0.0005f$ for a change of wave-length of 600 tenth-meters on each side of the minimum focus. For the three-prism spectrograph having a field of 4° (total) the permissible range of focus, as stated above, is about ¼ mm or $0.00023f$ on each side of the point of exact collimation. By setting the slit at a point about this distance outside the minimum focus we can therefore collimate the rays with the required degree of accuracy over a maximum total range of nearly 1,200 tenth-meters.[1] In this case, in order to *maintain* the accuracy of collimation over the entire range we must refocus the collimator for every change in temperature. On the other hand, if we set the slit at the exact focus of the collimator for any given wave-length and given

[1] This is a little less than the results claimed by Hartmann with the Potsdam three prism spectrograph. ASTROPHYSICAL JOURNAL, **12**, 39.

temperature, t, the required accuracy of collimation will be maintained for that particular spectral region over a range of temperature Δt such that

$$\Delta t = \left[\frac{\delta}{dt}(z) - \frac{\delta}{dt}(f)\right] = 0.00023f \ ;$$

or, since $\frac{\delta}{dt}(z) = az$ and similarly $\frac{\delta}{dt}(f) = a'''f$,

$$\Delta t \cong \frac{0.00023}{a - a'''} \ . \tag{94}$$

For objectives of the usual crown and flint glass the second term of the denominator a''' is about 0.000025 for 1° C. For brass and steel the values of a are about 0.000018 and 0.000010, as already given. Hence we have for $\Delta t_{\max}$:

$$\Delta t_{\max} \cong 15^\circ \text{ C. for steel a tube}$$

$$\Delta t_{\max} \cong 30^\circ \text{ C. for a brass tube.}$$

In general the most satisfactory procedure is to limit the range of wave-lengths for which we attempt to secure simultaneous collimation so as to leave a certain range of focus for changes in temperature. If we content ourselves with collimation over a range of 600 tenth-meters (which in the case of most three-prism spectroscopes represents an angular field of about four degrees) we have left to take care of temperature changes a range of focus of about $0.00012f$, which permits a maximum temperature variation

$$\Delta t_{\max} \cong 7.5^\circ \quad \text{for a steel tube}$$
$$\Delta T_{\max} \cong 15^\circ \quad \text{for a brass tube}$$

without refocusing.

It is very evident from (94) as well as from our previous considerations with reference to the focusing of the view telescope that so far as avoiding the effect of temperature changes is concerned, brass is a much more satisfactory material than steel to use for telescope tubes. Indeed it is possible by using this metal in conjunction with suitable varieties of glass to obtain almost perfect equality between the temperature coefficient of

expansion a' and the corresponding temperature coefficient of focal length a''' so that the range Δt of (94) may be increased and the quantity $d\Omega$ of (63) reduced to zero. Two troublesome adjustments and errors are thus avoided. The more detailed results of this investigation will be given in a subsequent paper.

[*To be concluded.*]

RESEARCHES ON THE ARC-SPECTRA OF THE METALS.

VI. SPECTRUM OF MOLYBDENUM.

By B. Hasselberg.

(*Concluded from p. 319, Vol. 16, December 1902.*)

RESULTS OF OBSERVATIONS.

After thoroughly taking into account the above comparisons I have collected in the following catalogue the wave-lengths of those lines of the arc-spectrum molybdenum which must be ascribed to this metal as far as may be judged at present. It will not be suprising if there should be among them doubtless a few and perhaps many which further investigation will show to be foreign lines, in view of the richness of the spectrum; nor if the use of stronger currents should make it possible to increase considerably the number of lines characteristic of the metal. The corrections and additions resulting under such circumstances can, however, refer only to the faintest lines, and it may therefore be hoped that this catalogue represents all of importance in respect to the spectrum of the metal.

In regard to the arrangement of the catalogue nothing needs to be added to what I have said in my previous papers. It may only be recalled that as before the wave-lengths given are the means of the results of two wholly independent series of measures, made on different plates, the internal agreement of which may be designated as satisfactory in every respect. The deviations of the values of the two series from each other are as follows:

	Tenth-Meters		Tenth-Meters
In 330 cases	0.00	In 10 cases	0.06
In 462 cases	0.01	In 2 cases	0.07
In 236 cases	0.02	In 3 cases	0.08
In 93 cases	0.03	In 3 cases	0.09
In 45 cases	0.04	In 1 case	0.10
In 16 cases	0.05		

Hence the cases in which this deviation exceed the value of 0.02 constitute only about 14 per cent. of the whole. The probable error of a wave-length therefore would in general not reach the amount of 0.02, an estimate which is shown to be practically correct by other circumstances as will be shown below.

Mo λ	R.	*i* *Mo*	*i* $\odot$	Remarks	*Mo* λ	R.	*i* *Mo*	*i* $\odot$	Remarks
3463.78					.....	3532.72			
65.81		1.2			3534.83		1.2		Very diffuse.
66.98		2			37.41		3		*Mo?*
67.13		1.2			39.07		1.2		
68.02		2—			39.62		1		
68.70		1+			42.32		2.3		
69.39		2			42.92		1		
69.80		1			43.27		1+	?	
71.09		1.2			47.57		1+		
.....	3473.43				48.05		1+		
75.19		2			48.88		1		
76.15		2—		76.07 *Cu*	51.12		1.2		
79.60		2			52.57		1	2	*Mo?*
80.26		1.2			.....	3553.62			
81.95		1.2			54.35		2—		
82.55		2			55.58		1.2	2	Coinc. *Mo?*
84.05		2			57.63		1		
90.42		1			58.25		3		At edge of $\odot$
.....	3491.46								line 58.21
91.92		1			59.42		1		
92.05		1+			60.28		2		At edge of $\odot$
92.98		1							line 60.28
93.49		2—			62.26		2—	1	
3498.21		1+			63.30		2.3	1	
3504.55		2.3	2	Also *Fe* 04.56;	63.91		2—		
05.45		2+		*V* 3504.57	64.45		1.2		
07.16		1			66.20		2+	?	
07.45		1			66.57		1		
08.26		2.3			66.91		1		
10.93		1.2			70.63		1.2		
.....	3512.78				70.82		2.3		
13.86		1.2			71.42		1.2		
14.93		1+			.....	3573.54			
17.70		2			74.05		2.3	2.3	$\odot$ has 74.05?
18.35		2							73.97 *Fe*
21.17		1.2			74.63		1.2		
			1		75.78		1.2		
21.32		1.2			75.88		1.2		
21.56		2.3			76.35		1.2		
22.52		1.2			80.70		2	?	
24.76		2+			81.15		1		
25.11		2+			82.03		3.4		
26.08		1+	?		83.30		4+		
3531.44		1+			3584.42		1.2		

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
3585.74		1+		
87.02		2 −		
89.10		2		At edge of ⊙ line 89.05
90.47		1		
90.90		2+		
91.55		1.2		
94.73		1		
.....	3595.45			
95.71		2−		
95.87		2−		
96.54		1−		
3599.05		2		
3600.04		1.2		
03.10		2.3		
03.86		1.2		Between the ⊙ lines 03.92 *Ti* .83 *Cr*
04.24		2−		
04.73		1.2		
05.19		1+	?	
07.56		1		
08.52		2+		
10.80		1−		
12.15		2−		
12.62		2−		
13.55		2−		
13.80		1.2		} Overlie the diffuse *Cu* band 13.86
13.94		1.2		
....	3614.26			
14.42		3		
14.87		1		
15.32		1.2		⊙ has 15.34 (*Fe*). No coinc.
15.91		1.2		
17.01		2−	1−	
23.36		2+	2	At red edge of the *Fe* line 23.34. R gives 23.36
24.60		3		
24.77		1.2		
26.33		2.3	1+	Also *Fe*
28.50		1+		
28.80		1.2		
29.45		2.3		
3635.30		2	2	*Mo?* But separated from the ⊙ line 35.34 *Ti, Fe*

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
3635.57		2.3		At edge of ⊙ line 35.61 *Ti, Fe*
35.77		1.2		
.....	3637.40			
37.68		2−	?	⊙ has 37.69. Coinc?
38.35		2.3		
38.57		1.2		Also in *V*. Foreign line?
38.72		1.2		
40.76		2+		
41.08	}	2		
41.16				
42.37		2−		
47.03		1+		
48.75		2		
49.61		1.2		At edge of ⊙ line 49.65 (*Fe*)
50.75		1	?	
51.48		2		
53.75		1		
54.73		2+	?	Diffuse. *Mo?* ⊙ has 54.81 *Fe* .74 *Ti* 54.6 *Cu*. Very diffuse
55.21		1.2		Diffuse. *Mo?*
57.53		2.3		⊙ has 57.56 *Fe*
58.50		1		
.....	3658.69			
59.51		3.4		
61.08		2−		
61.24		1.2		
61.91		2+	?	
63.14		2	?	
63.83		1+	?	
64.45		2−		
64.98		3	1	
66.87		2+		⊙ 66.91 *Fe*
68.63		1.2	?	
69.50		2.3	1	
72.97		3		
73.38		1.2		
75.54		2		
76.15		1.2		*Fe* has 76.11 separated
76.40		2+		
.....	3676.70			
77.83		2−		⊙ has 77.83—? .76 *Fe*
79.39		1.2		
80.36		1+		
80.75	}	3.4		On each side of ⊙ line 80.80
3680.85				

Mo λ	R.	*i* *Mo*	⊙	Remarks
3681.69		1.2		
81.88		2—		
82.12		1		
84.48		1.2		
86.27		2—		
86.72		2—		
87.12		1		
88.12		1		
88.45		2—		
89.13		2		
90.30		1		
90.72		2.3		
.....	3691.45			
92.24		1.2	?	
92.79		2—	2	*Mo?* ⊙92.79 *Fe*
93.52		2—		
95.09		3.4		
96.18		1.2	1	*Mo?* 96.17 *Fe*
3698.69		1.2		
3700.15		1.2		
01.67		1		
02.33		1+		
02.67		2		By the *Fe* line 02.63
.....	3704.60			
05.57		1+		
07.35		1.2		
08.73		2—		At edge of ⊙ line 08.79
10.32		1.2		
11.68		1.2		
12.22		1.2	?	Coinc.?
13.64		2—		
14.73		1.2		
15.83		2		
16.27		2	1	
17.05		1+		
18.66		1+		
19.71		1+		
19.87		1+		
20.42		2—		
22.50		1—		
23.70		1.2		
24.00		1+		
.....	3724.53			
25.75		2+		
26.45		2		Dbl. Center. $\Delta\lambda=0.09$
27.86		3		R. gives ⊙-27.78 (*Fe*). Wholly separated
3728.50		1.2		

Mo λ	R.	*i* *Mo*	⊙	Remarks
3728.70		2		
30.75		1		
32.91		3	2	Double. Center
33.22		2—		
33.59		1+		
34.56		1.2		
35.80		1.2	1—	Coinc?
36.36		1+		
38.10		2+		
.....	3740.20			
40.97		1+		
42.48		2.3		
43.98		1+		
44.55		2—		
45.12		1.2		
47.37		2—	1	
48.66		2+		
51.38		2+		
52.12		1—		
55.31		2		
55.68		1		
56.02		1.2		
58.70		2+		
59.80		1.2	1	
.....	3760.68			
61.07		2—		
61.93		2—		
62.27		1.2		
63.52		2		Also *Mn.*(63.-51) Coinc?
64.20		1		
64.60		1.2		
65.21		1+		
65.40		2—		
65.92		1.2		
66.58		1		
67.90		1+		
68.78		1+		
68.92		1.2		
70.66		2.3		
72.11		2+		
72.99		2+		
76.27		1+		
76.73		1+	1.2	
77.90		1.2		
.....	3778.20			
79.92		2+	?	⊙ has a faint pair. Coinc?
80.78		1—		
81.75		2.3	2	
82.35		1.2		
82.86		1		
85.19		2		
3785.67		1.2		

Mo λ	R.	i Mo	i ⊙	Remarks
3786.54		1		
88.42		2		
94.60		2−		
95.48		1+		
96.19		1.2		
96.45		1+		
97.20		1.2		
97.46		2−		
3798.39		10	1	Intense, broad, diffuse, reversed, with a fine absorption line; coincides with the ⊙ line
3800.28		1+		
.....	3801.82			
02.00		2.3		⊙01.98. Coinc?
02.35		1+		
04.70		2		
06.15		2		
07.82		1		
08.04		1		
08.79		1.2		
10.31		1+		
10.99		1		
11.56		1.2		Sharp
12.63		2		Sharp. Faint companion to violet
14.64		1+		
15.24		1.2		
17.37		1−		
18.83		2−		
19.98		2.3		
21.09		1.2		
21.82		1		
22.14		1		
23.17		2+		⊙ has a faint band here
.....	3823.65			
24.34		1.2		
24.94		1.2		
25.50		1.2		
25.63		1+		
26.85		2.3		
27.33		2+		
29.04		3−		
29.95		1.2		
30.08		1+		
30.22		2−		
3830.98		2−		Between ⊙ lines 31.00 } *Fe* 30.90 }

Mo λ	R.	i Mo	i ⊙	Remarks
3831.25		1.2		
31.95		1+		
32.26		2		
33.92		3		
34.82		1.2		
35.15		1.2		
35.49		2−	1.2	Coinc. ⊙ line diffuse
39.65		1+		
40.72		1+		
.....	3843.40			
44.09		1.2		
46.12		2+	2	Coinc. ?
46.36		1.2		
47.41		2+	2	Faint ⊙ group. Coinc.?
48.45		2+	2	Coinc.? R. gives ⊙ 48.43.—48.48 (*Ti*) separated from *Mo*
49.95		1+		
51.57		1+		
52.17		2−		
55.09		1		
56.15		1+	1.2	*Mo?*
.....	3863.53			
3864.25		10	1	Broad, diffuse, rever'd, with fine absorption line. R. gives 64.25 *Mo—C*
66.87		1+		
69.25		2.3	?	⊙ has a group here
70.62		1.2		
70.77		1.2		
73.30		1.2		Weak. At edge of ⊙ line 73.25 (*Co*)
74.34		1.2	?	Coinc.? 74.32 (*Ti*). Intensity variable? Foreign line?
79.20		1+		
.....	3885.66			
86.98		2.3		⊙ has 86.94 (*Cr*), separated
87.87		1		
3888.15		1−		

Mo λ	R.	*i* *Mo*	⊙	Remarks	*Mo* λ	R.	*i* *Mo*	⊙	Remarks
3888.36		2—			3938.88		1.2		
89.06		2—	?	Coinc ?	39.30		1+		
90.88		1.2			39.65		1+		
93.50		1+		Between the	40.50		1		
				⊙ lines	.....	3941.02			
				93.54—*Fe*	43.19		3		
				.45 *Co?*	43.66		2—	1+	*Al.* 44.10 dis-
96.55		1.2							tinctly faint-
97.05		1.2							er than 61.57
3897.68		1+			45.41		2—		⊙ has 45.47
3900.40		1—							(*Co*) separa-
00.87		1							ted from
01.95		2.3							*Mo*
.....	3902.40				47.00		1		
03.07		10	5	Extremely	47.33		2—		
				strong, dif-	50.40		1		
				fused, revers-	51.14		2—		
				ed with fine	51.49		1+		
				absorp'n line	51.70		1		
07.10		2—	1	*Mo?*	54.08		2		
08.42		1.2	1.2	*Mo?*	55.66		2—		
09.92		1.2			58.76		2—		
11.24		1.2		Diffuse	59.03		1		
12.10		1.2	1.2	*Mo?* 12.13	59.83		1+		
				Cr?	60.12		1+		*Al.* 61.57
13.52		1.2							strong
15.60		1			.....	3960.42			
16.62		1+			63.68		1.2		
17.09		2—			64.14		2—		
17.70		2+	?	Coinc ?	65.89		1.2		
17.95		2			66.40		1.2		
20.25		1—			68.91		2—		
21.09		1+			69.17		1		
22.49		2—			71.54		1.2		
23.91		2			73.10		1+	?	
.....	3924.67				73.92		2+		
24.78		1			74.09		2		
26.00		1			78.08		2—		
28.45		1.2	?	Coinc. ?	79.40		2—		
28.86		1.2			80.37		2		
28.95		1.2			80.87		1.2		
30.35		1.2			81.80		1.2		
31.57		1.2			......	3981.92			
34.41		1.2		The *Ca* lines	82.22		2		
				H and K	84.92		1+		
				occur with	85.88		1.2		
				slight inten-	86.45		2+	1	
				sity	91.55		2—	1	
35.13		2		Centr. of close	92.02		2—		
				double	93.22		1.2	1.2	Coinc. *Mo?*
35.33		1.2			94.06		2—	?	Perhaps Coinc.
36.30		1			94.79		1		
3936.89		1+		Also *Mn* For-	95.66		1+		Weak
				eign line ?	3998.45		2—		

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4000.55		2−		⊙ line 00.61 (*Fe*) between these two *Mo* lines
00.67		2		
03.62		1+		
......	4003.91			
05.86		1	2	Coinc. *Mo?* Also *V*. Other strong *V* lines are lacking
06.23		2		
06.85		1		
07.62		1		
08.21		1+		
09.53		2		
12.12		2−		Diffuse. *Mo?*
12.42		1		
12.68		1		
12.97		1−		
16.86		1+		
17.55		1.2		
......	4018.42			
19.32		1		
20.59		1.2		⊙ has 20.64−*Fe* .55 *Sc*
21.19		2		
25.64		1.2		
27.07		1+		
28.80		1.2		
31.06		1+		
31.60		1		
32.65		1.2		At red edge of ⊙ line 32.61 (*Fe* by R.; *V* by H.)
34.11		1	1−	
36.83		1+		
37.95		2−		
38.26		2	1−	
......	4040.79			
41.30		1.2		
43.05		2		
43.44		1		
43.91		1.2		
47.07		1		
47.56		1−		
47.75		1−		
49.78		1		
50.27		1+		
4051.35		1+		

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4056.18		2		56.22 *Cr* 56.13 *Fe*
57.61		1+		
57.77		1.2		Also *Ti* (57.76)
......	4059.08			
59.79		2−		
62.24		2.3		
66.52		2−	2	Coinc. *Mo?* 66.52 *Co*
67.88		1+		
70.05		3+		Hard to separate. Total intensity = 4
70.17		2		
75.43		2	1	
75.72		2−		
76.35		2		
76.69		1.2		Between the ⊙ lines 76.80 .64
78.25		1+		
......	4080.00			
81.62		3		
81.94		2−		
84.54		3	?	Perhaps Coinc
86.16		2		At red edge of ⊙ line 86.13
89.90		1.2		
93.32		1+		
94.63		1+		
96.98		2+		
4098.91		2+		
4102.33		2.3	1	
03.94		1.2		
......	4104.29			
05.27		2+		At edge of ⊙ line 05.31 *V*
05.72		2−		
07.63		3	3	⊙ has 07.65 *Ce, Fe, Zr* separated from *Mo*. *V* has 07.64
08.30		1.2	1.2	R. gives ⊙ 08.29. *Mo?*
10.46		1		
10.88		1+		
12.29		1		
13.77		1+		
15 08		2−	2	*Mo?*
19.12		2		At edge of ⊙ line 19.05 (*Fe*)
4119.81		1+	1	

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4120.26		3		
22.55		1		
23.83		2		
24.72		2		
......	4126.34			
28.46		2+		
29.02		2−		
32.07		2−		
32.41		2−		Intensity variable. Is it *Mo?*
32.90		1		
33.18		1+		
35.37		1		
35.55		1+		
37.10		1+		
38.35		1.2		
38.72		1.2		
39.72		1+		
42.28		1+		
43.73		4		
......	4147.84			
48.88		1+		
49.14		2.3		
49.90		1		
52.07		2−		At edge of ⊙ line 52.11
55.47		2.2		
55.77		2.3		
57.59		2.3		
58.27		1+		Overlies a diffuse band at 58.10. *Cu?*
60.44		1+		
62.85		2.3		Does not coincide with ⊙ 62.83
64.26		1.2		
65.94		1		
66.47		1.2		
68.68		1.2		
70.01		2−		⊙ has 69.93 (*Fe?*)
70.55		1		
......	4171.07			
71.27		1.2		At edge of ⊙ line 71.21 (*Ti*)
71.65		1+		
75.32		1	1.2	
77.09		1.2	1+	
77.45		2		
78.45		2		
4178.72		1		

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4180.12		1.2		
80.69		1.2		
81.24		2		Center of close double
84.33		1		
84.59		1.2		
85.98		3	1	
86.97		2−	1+	
88.49		4		
......	4192.73			
94.20		1		
94.74		2.3		
4199.82		1		
4200.02		1		
00.76		1.2	1.2	
01.35		1+		
01.50		1.2		
02.42		1		
04.80		1.2		
06.00		2		
07.42		1+		
07.75		1+		
08.97		1		
09.84		1	1	
10.39		1		
11.23		2−		
.....	4213.81			
14.24		1.2		
17.02		1		
19.20		1+		
19.55		2		Appears to lie between the ⊙ lines 19.52 and 19.58. These lines belong to *Fe*
20.17		1		
22.59		1.2		
23.15		1.2		
24.10		1		
24.93		1		
25.10		1		
26.44		1.2		
32.75		3+		
33.68		1.2		
35.23		1.2		
....	4236.11			
39.25		2−		
39.37		2−		
40.26		2		Also *V*
40.48		2		
41.03		2.3		
4242.97		1.2		

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks	*Mo* λ	R.	*i* *Mo*	*i* ⊙	Remarks
4244.95		1.2		Diffuse.	4308.85		1		
				Double ?	10.58		2—	2.3	⊙ has 10.63
									.54
46.19		2.3		At edge of ⊙	12.98		1.2		
				line 46.25	13.16		1.2		
				(*Fe*). K. R.	13.74		1+		
				give 46.20	15.60		1		
					18.13		2.3		
50.87		1.2			18.46		1		
51.58		1			.....	4318.82			
52.03		2.3			22.17		2—		
52.69		1+			22.66		1+		
53.77		1+			24.72		1		
.....	4257.82				25.44		1		
58.85		1+		Between two	26.33		3		
				⊙ lines	29.50		1+		
60.52		1.2			29.82		1.2		
60.85		1.2		⊙ has 60.89.	30.27		1+		
				Separated	32.68		1+		
61.17		1			33.40		1		
61.63		1.2			34.65		1		
64.81		1+			35.00		2	1	⊙ has a faint
66.27		2—							triple; co-
68.25		2—	2	Coinc. *Mo ?*					incidence is
69.44		2.3	1						with the
72.24		1.2							middle com-
73.23		1.2		Sharp					ponent
74.22		1+			36.38		1+		
.....	4274.96				38.73		1		
75.86		1+	?		38.90		2—		
77.08		3			39.42		1+		
77.38		3	?	R. gives ⊙	40.02		1.2		
				77.38.	40.93		2—		
77.58		2	?	R. gives ⊙	41.61		2+		
				77.54 *Zr.*	42.16		1		
				Coinc ?	.....	4344.67			
79.19		1+			44.86		1.2		
80.17		1+	1.2	Coinc. *Mo?*	46.40		1+		
82.00		2—			49.41		1—		
84.77		3		Diffuse. *Mo ?*	50.53		3—		
87.26		2—			53.48		2—		
88.82		3+			54.88		1—		
89.56		2+		At edge of ⊙	57.50		1.2		
				line 89.50	62.20		1.2		
				(*Ca*)	62.87		1+	1.2	Coinc. *Mo?*
91.39		2—			63.21		1—		
92.34		3			63.82		1.2		
93.42		3			64.65		1.2		
94.07		3			64.76		1+		
.....	4295.91				64.90		1+		
4296.35		1.2			66.73		2		
4301.45		1.2			.....	4368.07			
04.20		1.2			69.23		2.3		
4305.10		2—			4370.33		1		

Mo λ	R.	*Mo* i	⊙	Remarks	*Mo* λ	R.	*Mo* i	⊙	Remarks
4372.31		1+			4449.92		3		
73.52		1+			52.77		1.2		
75.07		1+			.....	4454.95			
75.21		1.2			57.55		3.4		
76.87		1+			58.84		1.2		
80.47		2+			60.80		2—		
80.80		1.2		Sharp	64.96		3—		⊙ has 64.94 *Fe*
81.36		1+							separated
81.82		4			68.28		1+		
82.61		2—			68.46		3		
86.10		1+			71.85		1.2		
.....	4388.06				72.23		1.2		
88.49		1			73.37		2.3		
89.76		1			74.78		4		
91.71		1.2			75.82		2+		
92.32		1.2			.....	4476.22*			
94.49		1.2			85.16		2.3		
94.67		1.2			87.23		2+		
96.55		1			89.17		1.2		
96.83		2		Sharp	90.37		2		
97.02		1			91.46		3		
97.48		2+	?	Perhaps coinc.	92.00		1		
98.68		1			92.24		1		
4402.67		2—			94.27		1+		
03.07		2+			4499.62		2—		
04.71		1.2		Sharp	4501.44		2		⊙01.42 *Ti* sep
07.04		1+							arated
09.61		1			06.13		} 3		
10.15		2—	1	Coinc.?	06.22				
.....	4410.68				06.86		2		
11.76		2.3			.....	4508.45			
11.90		3			12.32		2.3		
12.96		2+			15.20		1.2		
17.40		1			15.36		2—	1	Coinc.?
20.91		1			17.30		3+	1.2	Coinc. 17.28
22.23		1.2							*Co*
23.24		1+		A line here also	17.58		2		
				in *Ni.* Not	18.61		1+		
				given by R.	22.37		2+		
23.79		2.3			24.53		3		
24.40		1			25.50		1—		
26.86		2.3			25.56		2+		
28.39		1			28.77		2.3	4	Diffuse. Coinc.
29.32		1							*Mo?* 28.80 *Fe*
.....	4433.39				29.59		2.3		Sharp
33.68		1.2		Sharp	.....	4533.42			
37.06		2—			34.63		2		Diffuse. *Mo?*
37.35		1+			35.00		2+		Diffuse; broad.
39.15		1+		Sharp					*Mo?*
42.37		2.3		Sharp	35.56		2	?	Probably a ⊙
43.25		2		Sharp					line
44.21		1+		Diffuse. *Mo?*	37.00		3+		
46.62		2—		Sharp	38.60		1		
4447.41		1.2			4539.84		1		

* Double 76.253, 76.185. Center.

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4541.75		2—		
53.00		1		
53.40		1.2		
53.52		1.2		
54.00		2	3.4	*Mo?*
.....	4554.21			
58.30		2.3	1	Coinc.
58.92		1.2		
59.94		1+		
60.32		2+		Sharp. At edge of ⊙ line 60.27 (*Fe*). K. R.: 60.33
67.57		1—		
67.87		2+		
69.21		1+		
70.30		2		Sharp
70.78		1+		Sharp
74.66		1+		
74.80		1		
75.36		1—		
76.05		1+		
76.70		3—		Sharp
77.97		1+		Sharp
78.06		1		
.....	4578.73			
79.92		1		
82.52		1+		
82.69		1+		
86.25		1+		
86.75		1+		
86.98		1+		
87.61		1		
88.33		1.2		Diffuse. *Mo?*
90.55		2		Diffuse. *Mo?*
92.40		1.2		Sharp
93.84		1		
95.35		3		
98.07		1+	1.2	Coinc. *Mo?*
98.44		1+		
4599.35		2—	?	Probably a ⊙ line
.....	4603.13			
4603.78		1—		
08.32		1		
08.90		1+		Sharp
10.07		3	1	Coinc.
11.03		1+		
11.36		2—		Sharp
14.94		1		
16.81		1+	1.2	Probably not *Mo*
17.82		1		
18.15		1+		Sharp
21.57		2.3		Sharp
4623.66		1.2		Sharp

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4624.44		2		Diffuse. *Mo?*
.....	4625.23			
26.67		3.4		26.74 *Mn* separated. *V* 26.67 similarly
27.70		2.3	1	⊙ line extremely faint. Coinc.?
30.20		2—		Sharp
32.75		1		
35.22		1		
41.12		1—		
41.78		1—		
42.90		1.2		
.....	4646.35			
48.02		2		Sharp
49.28		1.2		
51.25		2—		
52.47		2		Diffuse, probably not *Mo*
56.57		1		
57.67		1		
62.11		2.3		Does not coincide with ⊙ 62.15
62.95		3	1	
63.31		1+		
65.59		1+		
.....	4667.77			
69.00		1+		
72.11		3—		
73.24		1		
75.91		1—		
.....	4679.03			
81.24		1+		
81.82		1		
82.44		1—		
84.04		1.2		
84.54		1+		
86.01		2—		
86.28		2—		
88.41		2.3		Separated from 88.46 (*Fe*). R. gives 88.36 (*Fe*)
91.05		2—		
92.19		1+		
92.89		1—		
93.55		1—		
96.06		1+		
4696.71		1.2		
.....	4700.34			
4700.71		2—		

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4706.25		2		Also *Cr*
06.40		1+		
07.44		3.4	2.3	Rowland gives 07.46 (*Fe*). $\lambda_{Fe} > 07.46$? K. R. 07.52
08.43		3−		
10.16		1		
14.69		2−		
16.88		1+		
18.13		2.3		
19.08		2		
.....	4722.34			
23.27		1.2		
23.50		1		
25.55		1+		
29.36		3.3		
31.64		3.4		
34.34		1+		
35.51		1		
36.84		1+		
40.36		1		
40.58		1		
.....	4741.72			
49.06		1−		
49.35		1−		
49.61		1−		
50.60		2.3		Sharp
51.31		1		
53.56		1		
56.06		1		
58.71		2.3		
60.39		4		Faint lines near by. *Mo?*
.....	4764.11			
64.64		2		Sharp
73.47		1.2		
73.64		2+		
74.42		2−		
75.87		2.3		
76.54		3	1.2	R. gives ⊙ 76.55 (*Co*). Also in *V* as strong line, $i=3$
78.09		1		
83.16		2.3	1	Coinc.?
.....	4783.61			
84.64		1		
85.34		2.3		Sharp
4786.68		2		At edge of ⊙ line 86.73 (*Ni*). *V* has 86.70

Mo λ	R.	*i* *Mo*	*i* ⊙	Remarks
4787.83		1		
88.39		1+		
92.96		2		
93.60		2		Sharp
94.03		1.2		
94.81		1+		
96.75		2.3		Sharp
.....	4798.45			
4805.13		1+		
05.78		2−		
08.29		2−		At violet edge of the ⊙ line 08.32 (*Fe*)
08.68		1+		
11.28		2.3		
14.68		1+		
17.92		2−		
.....	4817.99			
19.47		3		
22.62		1+		
23.16		1−		
28.67		2−		Sharp
30.15		1+		
30.73		3		Very sharp
33.13		1.2		
34.16		2−		
35.98		1−		
38.35		1		
.....	4839.73			
39.82		1		
45.38		1+		
50.05		1−		
51.92		1		
58.44		1.2		
.....	4859.93			
60.28		1.2		
60.99		1		Faint companion to red
66.07		1−		
68.23		3		
69.43		2		
.....	4871.00			
75.73		1+		At edge of the ⊙ line 75.67 (*V*)
78.59		1.2		
86.70		1+		
89.44		1+		
94.65		1−		
.....	4896.62			
97.50		1+		
4899.81		1−		
4904.03		2.3		
4907.65		1+		

Mo λ	R.	i Mo	i ⊙	Remarks
4909.41		1+		
.....	4917.41			
25.08		1−		
26.42		2−		
26.65		2−		
31.42		1−		
33.30		2		
33.99		1		
.....	4936.02			
41.90		2+		
50.83		2.3		
52.20		1−		
56.83		1−		
.....	4957.78			
57.78		3		R. gives 57.78 *Fe*. The lines are separated, however, and λ > λ *Mo*. Probably it should be λ_{Fe} =57.88. Kayser and Runge have 57.87
64.42		1.2		
64.63		2−		
75.58		1		
76.23		1−		
79.32		2.3		
.....	4980.35			
4995.55		1−		
5000.13		2+		
.....	5002.98			
14.80		1+		
16.99		2.3		
20.07		1−		
.....	5025.03			
29.21		2		
30.96		2−		
39.12		1−		
.....	5044.39			
46.73		1		
47.90		2		
55.22		1.2		
58.30		1		
60.07		2.3		
62.76		1+		
.....	5064.84			
80.23		2.3		
81.49		1		
.....	5084.28			
84.47		1−		
5090.80		1		

Mo λ	R.	i Mo	i ⊙	Remarks
5091.17		1.2		
91.56		1		
92.40		1+		
92.96		1		
96.11		1.2		
96.85		2+		At edge of the ⊙ line 97.67
97.71		2.3		
5098.27		1.2		
5100.58		1−		
.....	5105.72			At edge of the ⊙ line 09.83 (*Fe*)
09.90		2+		
15.21		2		
15.86		1−		Sharp
17.18		1.2		
22.00		1		
.....	5123.90			
24.03		1+		
26.94		1		
35.17		1		
41.47		1+		
.....	5146.66			
48.65		1		
55.48		1−		
63.40		2+		
.....	5165.59			
67.98		2		
71.33		3		
73.14		3		In the shade of b_2
74.35		3		
5180.44		1.2		
.....	5186.07			
5200.37		2−		
00.97		1		
.....	5204.72			
12.08		1		
19.62		1.2		
.....	5225.70			
31.27		1+		
32.58		1−		
34.47		2−		
38.41		3		Companion to violet
41.09		3		
.....	5242.66			
43.01		2		
45.71		2−		
59.23		2+		
61.35		2−		
.....	5266.73			
72.00		1−		
76.50		1−		
5279.85		2−		

Mo λ	R.	*i* *Mo*	⊙	Remarks
5281.07		2+		
.....	5288.71			
92.30		1.2		
93.65		1		
5295.67		1.2		
5306.49		1−		
.....	5307.54			
14.13		2−		
18.20		1		
20.14		1		
24.70		1−		
27.35		1		
.....	5328.75			
55.12		2−		
55.76		1+		
56.70		2−		
60.76		4.5		
64.50		3.4		
67.30		2−		Diffuse
.....	5370.17			
72.63		1+		
88.94		1		
.....	5389.68			
94.75		2		Sharp. ⊙ has 94.91 } 84 } *Mn*
5397.63		1.2		
5406.64		1.2		
11.31		1−		
14.95		1−		
.....	5415.42			
17.64		1.2		
26.24		1		
27.14		1.2		
27.80		1		
31.27		1+		
.....	5434.74			
35.91		2+		
37.97		2.3		
39.95		1+		
47.86		1−		
48.78		1−		
50.73		2.3		
53.27				
.....	5455.75			
56.71		2+		
65.83		2−		
73.64		3		
76.18		2		
.....	5477.12			
88.91		1−		
5490.54		2		

Mo λ	R.	*i* *Mo*	⊙	Remarks
5492.43		2−		
94.06		2		
97.18		1.2		Diffuse
98.76		2−		Sharp
5499.77		1+		Diffuse
5501.78		2		Sharp
02.18		2−		
03.82		1.2		
06.75		6		
.....	5507.00			
11.77		1+		
17.73		1		
20.32		1.2		
20.93		1.2		
.....	5525.76			
26.81		2		
27.27		2		
32.00		1+		
33.26		6		
34.85		1		
39.67		2		Sharp
41.93		1+		Sharp
43.38		2+		Sharp at violet edge of ⊙ line 43.41 (*Fe*)
.....	5544.16			
44.78		2		Sharp
52.47		1+		
.....	5555.12			
56.55		2.3		
57.02		2		
62.74		1		
63.65		1		
64.34		2−		
68.88		2.3		
69.75		2		
70.69		6		
75.47		2+		
.....	5576.32			
89.02		2	2.3	Coinc.? At edge of ⊙ line 88.98 (*Ca*)
91.84		2−		
.....	5594.69			
5596.62		1.2		
5601.31		1.2		
08.90		2		
09.53		2+		
09.80		1		
11.20		3		
13.37		2		
.....	5615.88			
18.69		2−		
5619.03		1.2		

Mo λ	R.	*Mo* *i*	⊙	Remarks
5619.63		2—		
32.74		4		
.....	5634.17			
35.14		2.3		
42.05		1—		Near the ⊙ line 42.11 (*Ni*)
43.47		1—		
50.40		4		
51.54		1+		
52.12		1.2		
52.47		1+		
.....	5657.72			
64.65		1.2		
67.57		1.2		
72.35		1+		
73.92		2+		
74.77		2.3		
.....	5675.65			
78.18		2.3		
83.20		2+		
87.93		1+		
89.39		4.5		Companion to red
.....	5693.86			
94.64		1+		
95.10		1		
95.66		1—		
96.30		1.2		
98.53		2—		
5699.57		2+		
5702.39		1.2		
05.97		3		
08.28		1+		
.....	5711.31			
12.05		2		Near the ⊙ line 12.09 (*Ti*)
19.55		1—		
20.45		1—		
22.98		3.4		
29.03		2—		
29.77		2		
30.17		2—		
31.58		1—		
.....	5731.98			
34.32		2—		
35.55		1—		
38.40		1—		
39.93		1		
41.96		1		
47.08		1—		
5747.93		1+	1+	

Mo λ	R.	*Mo* *i*	⊙	Remarks
5751.67		4.5		
.....	5754.88			
57.80		1		
65.57		1—		
66.79		1—		
67.63		1—		
70.02		1+		
71.33		1+		
.....	5772.36			
74.85		1.2		
78.46		1		
79.65		2		
80.38		1		
80.96		1—		
83.54		2—		
85.99		1		
.....	5788.14			
5792.10		4		
5800.72		2—		
02.95		2+		
06.46		1—		
08.54		1+		
09.30		1		
.....	5809.44			
14.14		1		
15.76		1+		
16.00		1		
21.00		1—		
25.28		1		
25.50		1.2		
.....	5831.82			
35.87		1—		
40.25		1—		
.....	5848.34	...		
49.16		1.2		
49.99		2		
51.80		2		
58.52		4		
61.66		1—		
.....	5866.67			
69.05		1—		
69.57		2		
76.90		1		
81.85		1—		
.....	5883.07			
83.11		1—		
88.61		4		
91.89		1		
.....	5893.10			
5893.67		2		

SYSTEMATIC ERRORS IN THE MEASUREMENT OF SPECTRUM PLATES.

In the treatment of the photographic plates I have always followed the procedure of dividing the metallic lines into small groups, of from fifteen to twenty tenth-meters' range, and referring them to two lines of Rowland's list, simple if possible, from which the wave-lengths could be obtained with sufficient precision by linear interpolation; but, for the sake of controlling still further the values so obtained, a solar line is commonly also measured in every group, and its wave-length determined with the metallic lines must then also agree with the value given by Rowland. To show the accuracy with which this agreement was attained in these measures, I give the following table containing a series of such solar lines together with a comparison with Rowland's values:

H.	R.	H.– R.	H.	R.	H.– R.	H.	R.	H.– R.
3717.523	17.539	–0.016	4070.912	70.930	–0.018	4374.621	74.628	–0.007
29.954	29.952	+ 002	71.893	71.908	– 015	4395.198	95.201	– 003
44.253	44.251	+ 002	76.102	76.101	+ 001	4425.598	25.608	– 010
66.799	66.801	– 002	4098.327	98.335	– 008	4449.325	49.313	+ 012
86.300	86.314	– 014	4100.878	00.901	– 023	4571.261	71.275	– 014
3792.472	92.482	– 010	14.605	14.606	– 001	4598.286	98.303	– 017
3852.710	52.714	– 004	18.696	18.708	– 012	4636.025	36.027	– 002
73.900	73.903	– 003	36.675	36.678	– 003	4691.573	91.602	– 029
3892.073	92.069	+ 004	56.962	56.970	– 008	4772.998	73.007	– 009
3916.868	16.879	– 011	63.802	63.818	– 016	4810.710	10.724	– 014
33.822	33.825	– 003	4185.040	85.058	– 018	5234.787	34.791	– 004
41.897	41.878	+ 019	4202.197	02.198	– 001	5250.808	50.817	– 009
48.227	48.246	– 019	04.138	04.132	+ 006	5322.234	22.227	+ 007
52.099	52.103	– 004	22.363	22.382	– 019	5353.575	53.571	+ 004
68.611	68.625	– 014	25.605	25.619	– 014	5445.255	45.259	– 004
3996.118	96.140	– 022	48.381	48.384	– 003	66.582	66.609	– 027
4007.409	07.429	– 020	67.108	67.122	– 014	5497.723	97.735	– 012
10.298	10.327	– 029	4288.319	88.310	+ 009	5560.430	60.434	– 004
30.323	30.339	– 016	4307.914	07.907	+ 007	5638.477	38.488	– 011
45.977	45.975	+ 002	08.075	08.081	– 006	5662.737	62.744	– 007
50.820	50.830	– 010	37.193	37.216	– 023	5701.766	01.772	– 006
66.508	66.524	– 016	51.200	51.216	– 016	5763.200	63.218	– 018
			58.657	58.670	– 013	5816.558	16.601	– 043

The first fact which attracts attention in this table, aside from the generally very small amount of the differences H.– R., is that with few exceptions they are negative, and hence my wave-lengths are throughout too small. Accordingly, there is a systematic difference or a personal equation between myself and Rowland, the amount of which, –0.009 tenth-meters, is the mean of the above differences. If this is added, with the reversed sign, as a systematic correction to the differences H.– R., they

then take the character of accidental errors of observation, from which the probable error of a wave-length measured by me, referred to Rowland's system, comes out 0.007 tenth-meters, an accuracy which must be regarded as in fact very satisfactory.

It may be regarded as certain that in general an equal precision is not attained for the metallic line; but if we should in this case double the probable error, it would not then reach the value 0.02, as already remarked; or, in other words, in all probability the measurements are not inconsiderably more accurate than I have hitherto considered myself justified in assuming.

In regard to the origin of the above-mentioned systematic difference between my measures of the solar lines and those of Rowland—which difference, by the way, appears of exactly the same magnitude in the measures of the arc-spectrum of tungsten which I have just concluded—we can hardly assume that it is to be sought in other than purely physiological peculiarities, which cause a certain dependence of the setting of the thread of the microscope on the spectral line upon the direction from which the settings are made. In my measurements this direction is always apparently from left to right in the field of view of the microscope, which is also the direction of decreasing wave-lengths. In order to get more accurate knowledge as to how far this suspicion is justifiable, I measured a number of solar lines used in connection with the tungsten spectrum after reversing the direction of the plate, hence from violet toward red; I thus obtained values which differ from the previous ones only slightly, but nevertheless systematically. These measures are contained in the first two columns of the following table, which gives also the differences of the two series of measures, their mean, and a comparison with the corresponding determinations by Rowland.

An examination of the third column of this table leaves us hardly room to doubt that there is a difference, distinct although very small, between the two series, amounting in the mean to 0.006 tenth-meters. In the first position of the plate the wave-lengths are accordingly too small by 0.003 tenth-meters, in the second position just as much too large. We have therefore

I	II	II – I	$\frac{I+II}{2}$	R.	H. – R.
4015.753	763	+0.010	758	760	−0.002
25.962	972	+ 010	967	972	− 005
52.078	083	+ 005	080	070	+ 010
70.409	419	+ 010	414	431	− 017
4087.243	245	+ 002	244	252	− 008
4106.583	576	− 007	580	583	− 003
25.771	781	+ 010	776	776	000
42.017	025	+ 008	021	025	− 004
63.796	815	+ 019	806	818	− 012
4182.916	914	− 002	915	922	− 007
4201.078	075	− 003	077	089	− 012
20.493	506	+ 013	500	509	− 009
39.501	511	+ 010	506	525	− 019
58.475	490	+ 015	482	477	+ 005
74.332	341	+ 009	336	348	− 012
4299.140	147	+ 007	143	149	− 006
4307.908	917	+ 009	912	907	+ 005
08.078	080	+ 002	079	081	− 002
13.029	038	+ 009	033	034	− 001
27.269	263	− 006	266	274	− 008
44.439	451	+ 012	445	451	− 006
68.054	067	+ 013	060	071	− 011
4379.377	394	+ 017	385	396	− 011
4400.548	546	− 002	547	555	− 008
17.858	868	+ 010	863	884	− 021
43.358	356	− 002	357	365	− 008
56.777	780	+ 003	778	794	− 016

obtained only a partial explanation of the systematic difference of my wave-lengths from Rowland's, since a systematic difference of −0.006 still remains after the values have been freed from the effect of the direction of measurement. If we could assume a personal error, of a similar sort but in opposite direction, for the measures of Rowland, everything systematic in the mutual differences of our measures would be eliminated, and they could be regarded purely as errors of observation. There seems to me to be nothing *a priori* against such an assumption.

Inasmuch as the probable error of the wave-lengths I determined in the solar spectrum amounts, as above stated, to ± 0.007 tenth-meters, or almost exactly the mean difference of series I and II, we might question the validity of the assumption of the slight effect of the direction in which the settings were made on the solar lines. The entirely similar appearance of the objects set upon—the line to be measured and the normal line—would

seem to exclude every sort of systematic difference in the mode of making the setting. However, since the differences of the two series are, with few exceptions, in the same sense, as appears from the third column, it seems to me we can hardly deny the actual presence of such an effect, although for one I am unable to produce any sufficiently satisfactory explanation for it.

Accordingly, if the observation of such similar objects as the solar lines appearing on a dark background gives evidence of an effect, however small, of the direction in the which the setting is made, we should certainly expect in advance a similar and more marked effect when the lines of the metallic spectrum were to be referred to those of the Sun. We should much sooner assume a difference in the mode of setting according to the position of the image in the case of setting the thread first on the bright solar line on a dark field and then upon the dark metallic line in a bright field, and this has been fully confirmed. In order to investigate the question further, I repeated the measures with the plate reversed for certain groups of lines of tungsten, which I had already measured in the usual direction of decreasing wave-lengths from left to right in the field of view of the microscope. The following table, in which the wave-lengths obtained in the two positions of the plate are given under the heading I and II, shows the results of these measurements:

I	II	II—I	I	II	II—I
4411.871	.834	—0.037	4425.064	.030	—0.034
12.347	.343	— .004	26.087	.052	— .035
13.173	.142	— .031	27.533	.508	— .025
14.042	13.987	— .055	28.663	.607	— .056
15.233	.246	+ .013	32.379	.314	— .065
15.891	.845	— .046	35.903	.874	— .029
18.609	.584	— .025	37.070	.070	.000
18.967	.934	— .033	38.469	.441	— .028
19.429	.387	— .042	39.904	.845	— .059
20.627	.608	— .019	42.004	41.976	— .028
21.168	.135	— .033	44.650	.592	— .058
22.010	21.958	— .052	45.324	.297	— .027
			49.182	.164	— .018

We see that here also a systematic differance of the appreciably larger amount of 0.033 tenth-meters appears, the wave-lengths being measured too large by 0.016 in the first position,

and as much too small in the second. In the first position of the plate the left side in the field of view was toward larger wave-lengths, and hence it follows that in the two cases the setting of the thread on the metallic lines was systematically somewhat too far toward the left in respect to the settings on the solar lines. This physiological difference in the *Auffassung* of the two different kinds of spectral lines is so pronounced as to leave no doubt as to its reality. This systematic correction or personal equation in my measures of metallic spectra is decidedly larger than that just discussed in the case of the solar lines, but it is also only of about the same magnitude as the probable error of the wave-lengths of the metallic lines, and consequently of no great significance. It has not been taken into account in the above catalogue of the lines of molybdenum, since in its determination the plates of tungsten and not those of molybdenum were employed.

The personal equation in the measurement of photographic plates of spectra here described has also been noticed elsewhere. In his measurements of stellar plates for the determination of radial velocities, Reese,[1] of the Lick Observatory, found that his settings on the dark lines of the metallic comparison spectrum were systematically somewhat too far toward the right in the field of view of the microscope in comparison with the settings on the light lines of the star, producing a systematic correction of about 1 km. This is evidently the same phenomenon that occurs in my measurements, except that it occurs in the opposite sense, which is not at all surprising in view of the wholly personal character of the phenomenon. We may remark, as an odd coincidence, that the magnitude of the personal equation is the same in the two cases. If the above measurements had been made for determining radial velocities, a variation in wave-length of 0.016 tenth-meters would easily be found to correspond in the region of spectrum considered to a change of velocity of 1.08 km.

COMPARISONS WITH THE INVESTIGATIONS OF THE SPARK SPECTRUM BY EXNER AND HASCHEK.

It has already been indicated above that, while the arc-spectrum of molybdenum has hitherto remained practically unknown,

[1] *Lick Observatory Bulletin* No. 15; ASTROPHYSICAL JOURNAL, **15**, 142, 1902.

its spark spectrum has recently been subjected to a thorough examination in the portions affecting common photographic plates by Exner and Haschek in the course of their similar investigations of all the chemical elements. Since it was the primary purpose of these investigations to establish a basis for future mineralogical researches, the greatest accuracy of the determinations of the wave-lengths was less important than the most rapid and convenient possible method of operation. Accordingly the authors determined the wave-lengths by simply projecting a thirtyfold enlargement of the plate upon a scale graduated to half-centimeters, on which, after the adjustment of the standard lines, the wave-lengths of the remaining lines were obtained by a single reading. With the enlargement employed, the 5 mm scale divisions represent tenth-meters, and accordingly the reading had to be accurate only to 0.5 mm in order to attain the accuracy of 0.1 tenth-meters originally sought for. It might, however, seem questionable in how far this method of observation can be protected from systematic errors. In order to obtain definite data on this point, I have, for the 600 lines of molybdenum common to our catalogues, taken the means of the differences of the wave-lengths by tens, and have thus obtained the following mean deviations for the portions of spectrum given:

λ	H. − E.H.	λ	H. − E.H.	λ	H. − E.H.	λ	H. − E.H.
3510	+0.09	3740	+0.07	3994	+0.14	4318	+0.03
3530	+ .07	3757	− .02	4020	+ .16	4333	+ .01
3553	+ .09	3770	− .02	4040	+ .13	4357	+ .05
3568	+ .12	3785	+ .02	4056	+ .04	4381	+ .05
3585	+ .12	3800	− .01	4090	+ .01	4400	− .08
3603	+ .06	3812	+ .05	4115	+ .03	4425	− .09
3617	+ .02	3825	+ .03	4134	− .03	4450	− .07
3630	+ .02	3837	+ .02	4152	− .06	4475	− .10
3650	.00	3856	+ .02	4177	+ .12	4502	− .05
3662	+ .04	3875	+ .03	4200	+ .08	4529	− .02
3677	+ .07	3900	+ .06	4225	+ .05	4558	− .02
3692	+ .08	3918	+ .07	4248	+ .08	4565	.00
3707	+ .11	3935	+ .11	4265	.00	4607	+ .06
3720	+ .08	3955	+ .10	4280	− .05	4640	+ .15
3730	+0.11	3980	+0.13	4295	−0.05	4684	+ .19

We may safely conclude from these figures that the accuracy sought is in general attained; but a certain periodicity is revealed

in the way the figures run, which would confirm the suspicion expressed above if there were not other circumstances which would appear to throw the systematic element of the deviation rather upon the plates themselves than upon the adjustments of the enlarged images on the scale and the reading. In the course of their further investigations Exner and Haschek have, in fact, found that in the method employed at first of photographing the comparison spectrum in juxtaposition to the spectrum under investigation by successive exposures, there occurred small displacements of the two spectra in respect to each other which were beyond checking, due to an insufficient stability in the mounting of the spectrograph. The injurious effect of these displacements could be avoided only by photographing the iron comparison spectrum simultaneously with the spectrum under investigation as an impurity of it. With the use of such plates new wave-lengths were obtained by projection in the same manner as before for the stronger lines of the metals previously investigated, including molybdenum. The following table gives the comparison of these new wave-lengths with mine:

H.	E.H.	Δ	H.	E.H.	Δ	H.	E.H.	Δ
3524.76	3524.70	+0 06	3786.54	3786.43	+0.11	4269.44	4269.43	+0.01
37.41	37.40	+ .01	98.39	3798.35	+ .04	77.08	77.00	+ .08
85.74	85.78	− .04	3864.25	3864.25	.00	77.38	77.43	− .05
3596.54	3596.51	+ .03	3903.07	3903.07	.00	79.19	79.15	+ .04
3612.15	3612.27	− .12	41.62	41.60	+ .02	88.82	88.78	+ .04
14.42	14.43	− .01	3986.45	3986.32	+ .13	92.34	92.25	+ .09
35.30	35.29	+ .01	4062.24	4062.20	+ .04	4293.42	4293.33	+ .09
51.48	51.30	+ .18	70.11	70.05	+ .06	4326.33	4326.30	+ .03
58.50	58.48	+ .02	81.62	81.60	+ .02	63.82	63.75	+ .07
80.80	80.82	− .02	4084.54	4084.51	+ .03	4381.82	4381.77	+ .05
84.48	84.32	+ .16	4107.63	4107.61	+ .02	4411.90	4411.82	+ 08
88.45	88.42	+ .03	19.81	19.75	+ .06	33.68	33.62	+ .06
3692.79	3692.82	− .03	20.26	20.25	+ .01	49.92	49.91	+ .01
3702.67	3702.70	− .03	22.55	22.45	+ .10	57.55	57.50	+ .05
17.05	17.10	− .05	43.73	43.70	+ .03	74.78	74.72	+ .06
42.48	42.45	+ .03	85.98	86.00	− .02	4491.46	4491.43	+ .03
44.55	44.55	.00	4188.49	4188.50	− .01	4517.30	4517.30	.00
55.68	55.63	+ .05	4209.84	4209.82	+ .02	17.58	17.50	+ .08
3782.35	3782.17	+ .18	32.75	32.80	− .05	24.53	24.50	+ .03
			44.95	44.90	+ .05	4537.00	4536.98	+ .02
			4250.87	4250.82	+ .05	4610.07	4610.00	+ .07

We see that, in fact, not only a far better agreement is attained, but that also the former periodic variation of the differences can no longer be recognized. It therefore appears probable that these variations were produced by the above-mentioned small mutual displacements of the spectra, which occurred

first toward one side and then toward the other in the plates of the different portions of the spectrum; and it would appear accordingly that no systematic errors of *this sort* are to be feared for the direct method of reading in itself. The prevalence of a positive sign of the differences indicates, however, a small systematic deviation averaging +0.03 tenth-meters,[1] which is doubtless of personal origin, and may be partially explained by the above-mentioned personal equation affecting my measures. If we apply this as a systematic correction we get as the probable deviation:

$$\text{H.} - \text{E.H.} = \pm 0.032 \text{ tenth-meters,}$$

whence, on the assumption of a probable error of ± 0.015 in my wave-lengths, there would follow a probable error of ± 0.027 for the wave-lengths of Exner and Haschek. The accuracy is therefore almost the same. This result is not a little surprising, and if it should be confirmed by further comparisons of the tables of Exner and Haschek with measured wave-lengths, it would argue for the adoption of their method of direct reading, at least in cases where the occasional occurrence of isolated larger deviations was not of importance in comparison with the great importance of rapid work.

RELATIONS BETWEEN THE SPECTRUM OF MOLYBDENUM AND THAT OF THE SUN.

As a preliminary result of his comparisons of metallic spectra with the solar spectrum, Rowland published two lists of those metals whose presence in the absorbing stratum of the Sun may be regarded as certain. In these lists the metals are are arranged, first in the order of intensity of the corresponding solar lines, and second, in the order of the number of observed coincidences. In both cases molybdenum occupies a position near the middle. Since in the latter list molybdenum precedes magnesium, for which over twenty coincidences were observed, molybdenum should be represented by at least an equal number of coincidences in the general solar spectrum. But, if we search Rowland's list of solar spectrum wave-lengths for coincidences

[1] The lines at $\lambda 3651.30$ and 3782.17, which showed the unusual difference of 0.18 tenth-meters, were excluded in taking the average.

with molybdenum, we find only seven such, whence it must be assumed that Rowland wished to include only the very strongest lines in this list, expressly designated as provisional, and that he considered a further examination of the reality of the coincidences necessary for the less conspicuous lines. This view seems the more probable on account of the fact that with few exceptions the ultimate decision of this question of coincidence is rendered extremely difficult by the very slight intensity of the solar lines concerned in comparison with the molybdenum lines.

The above-mentioned lines of molybdenum occurring in Rowland's list of solar lines are the following:

λ	Intensity in ⊙ (Rowland)	λ	Intensity in ⊙ (Rowland)
3132.75	1	3304.37	1
3170.45	2	3798.40	0
3194.09	000	3864.25	1
3264.53	0		

Only the first two of these lines fall within the limits of my previous observations of the arc-spectrum of molybdenum. They appear on my plates with a very conspicuous intensity, far surpassing all the other lines of the spectrum, and they are greatly broadened and contain narrow, sharp absorption lines which can be observed to coincide beyond a doubt with two fine solar lines. The line at λ 3903.07, which together with the two just-mentioned lines constitutes the most conspicuous element of the whole spectrum, is of almost the same intensity and otherwise of similar appearance. Rowland gives in the solar spectrum at λ 3903.09 a very strong line, ascribed to *Fe* and *Cr*, of which the solar analogue of the *Mo* line, if present, is probably a faint and very close companion not separable from it. The intensity of the solar line given by Rowland in itself makes improbable any relation to the molybdenum line, and this is further confirmed by the fact that on my double exposures to the molybdenum and iron spectra a separation of the two in the sense $\lambda_{Mo} < \lambda_{Fe}$ was suspected on account of the reversal of the two lines. If

the *Mo* line in question is represented in the solar spectrum like the two other lines, as can hardly be doubted, then the corresponding solar line must be so closely blended with the line at λ 3903.09 that they could not have been separated by Rowland. We have in this fact without doubt the reason why Rowland left this strong line of molybdenum unmentioned in his list of the solar lines.

I know nothing as to the intensity of the remaining lines attributed by Rowland to *Mo*. With the exception of the last, they are all lacking from Exner and Haschek's list[1] of the stronger lines of the arc-spectrum, while the three principal lines of the arc-spectrum also appear there as the strongest lines of the entire spectrum.

The catalogue of wave-lengths shows that the number of more or less certain coincidences with solar lines which I have been able to observe in this investigation of the *Mo* spectrum, aside from those already mentioned, is very limited. If we consider only those cases in which on account of the greater intensity of the *Mo* lines, the suspicion of contamination by a foreign metal may be regarded as excluded, we obtain the following summary:

λ	*i* *Mo*	*i* $\odot$	Sun λ	(Rowland) *i*	
3563.30	2.3	1	3563.30	000	
3626.33	2.3	1+	3626.33	1	*Fe*
3664.98	3	1	3664.97	000	
3669.50	2.3	1	3669.54	00	
3969.25	2.3	?	3969.29	0	*Cr, Co*
4084.54	3	?	4084.58	0	
4102.33	2.3	1	4102.32	0	*V*
4185.98	3	1	4185.94	0	
4269.44	2.3	1	4269.45	0	
4277.38	3	?	4277.38	1	
4517.30	3	1.2	4517.32	0	*Co*
4558.30	2.3	1	4558.29	0	
4610.07	3	1	4610.09	0	
4627.70	2.3	1	4627.73	0	
4662.95	3	1	4662.93	0	
4776.54	3	1.2	4776.55	0	*d? Co*
4783.16	2.3	1	4783.17	00	

With the exhibit of this table, in connection with what has been

[1] *Wien. Sitzungsber.*, **106**, 1897.

said above as to the three principal lines of *Mo*, the presence of the metal in the general absorbing layer of the Sun may be regarded as proven. My investigations thus far do not permit a decision as to how far the remaining lines of *Mo* of greater intensity are represented in the general solar spectrum. The following comparison of these strong *Mo* lines with Rowland's solar spectrum indicates with considerable probability that in several cases this is true. In making this comparison I have assumed the possibility of a coincidence only in cases where the difference of wave-lengths does not exceed 0.05 tenth-meters.

Mo λ	*Mo* *i*	Sun (Rowland) λ	Sun (Rowland) *i*	Remarks
3508.26	2.3	3508.23	000	
21.56	2.3	.....		
37.41	3	37.44	0000	
42.32	2.3	....		
58.25	3	58.21	1	
70.82	2.3	70.83	0000	
3582.03	3.4	3582.08	1	
3603.10	2.3	3603.11	0000	
14.42	3	14.45	0000	
24.60	3	24.60	0000	
29.45	2.3	29.49	000	
38.35	2.3	38.38	1	
57.53	2.3	57.56	1	*Fe*
59.51	3.4	.....		
72.97	3	72.94	0000	
80.75 } 80.85 }	3.4	80.80	3	no coinc.
90.72	2.3	90.73	0000	
3695.09	3.4	3695.04	0000	
3727.86	3	3727.83	1	
32.91	3	32.89	2	
42.48	2.3	.....		
70.66	2.3	70.67	000	
3781.75	2.3	3781.75	1	
3802.00	2.3	02.05	00	*Mn*
19.98	2.3	19.94	00	
26.85	2.3	26.84	0	
29.04	3—	.....		
33.92	3	33.92	0	
3869.25	2.3	.....		
3901.95	2.3	.....		
3943.19	3	.....		
4062.24	2.3	4062.20	00	
70.05	3+	70.05	000	
4081.62	3	4081.58	00	
4107.63	3	4107.65	5	*Ce, Fe, Zr*
20.26	3	.....		
43.73	4	.....		

Mo λ	*Mo* *i*	Sun (Rowland) λ	Sun (Rowland) *i*	Remarks
4149.14	2.3	....		
55.47	2.3	55.48	00	
55.77	2.3	55.80	00	
57.59	2.3	57.59	00	
62.85	2.3	62.83	1	no coinc.
88.49	4	88.48	00	
4194.74	2.3	4194.78	00	
4232.75	3+	4232.76	00	*V*
41.03	2.3	.....		
46.19	2.3	46.18	0	
52.03	2.3	52.04	000	
77.08	3	.....		
88.82	3+	.....		
92.34	3	.....		
93.42	3	.....		
4294.07	3	4294.08	000	
4318.13	2.3	.....		
26.33	3	.....		
50.53	3—	4350.55	00	*Ba?*
69.23	2.3	4369.25	00	
4381.82	4	.....		
4411.76	2.3	4411.75	00	
11.90	3	11.88	00	
23.79	2.3	23.75	000	
26.86	2.3	26.84	000	
42.37	2.3	.....		
57.55	3.4	.....		
68.46	3	68.46	00	
73.37	2.3	73.38	000	
74.78	4	74.74	00	
4485.16	2.3	85.12	000	
4491.46	3	.....		
4506.13 } 06.22 }	3	4506.09 06.26	000 00	*Ba*
12.32	2.3	.....		
24.53	3	24.57	1	*Mn*
29.59	2.3	.....		
37.00	3+	.....		

Mo λ	*i*	Sun (Rowland) λ	*i*	Remarks	*Mo* λ	*i*	Sun (Rowland) λ	*i*	Remarks
4576.70	3—	76.69	00		5097.71	2.3	5097.67	0	no coinc.
4595.35	3	4595.39	00		5171.33	3	.....		
4621.57	2.3	.....			73.14	3	.....		
27.70	2.3	4627.73	0		5174.35	3	.....		
62.11	2.3	62.15	1	*Fe?*	5238.41	3	5238.42	000	
62.95	3	62.93	0		5241.09	3	5241.04	000	
72.11	3—	72.09	000		5360.76	4.5	.....		
4688.41	2.3	.....			5364.50	3.4	.....		
4707.44	3.4	4707.46	5	*d? Fe*	5437.97	2.3	5438.00		
08.43	3—	08.46	000		50.73	2.3	.....		
18.13	2.3	.....			5473.64	4	.....		
29.36	2.3	29.38	0000		5506.75	6	5506.72	000	
31.64	3.4	31.65	4	*Fe?*	33.26	6	33.25	000	
50.60	2.3	.....			56.55	2.3	.....		
58.71	2.3	.....			68.88	2.3	68.92	0000	
60.39	4	60.40	0000		5570.69	6	.....		
75.87	2.3	.....			5611.20	3	.....		
85.34	2.3	.....			32.74	4	.....		
4796.75	2.3	.....			35.14	2.3	.....		
4811.28	2.3	4811.24	00	*Ti*	50.40	4	5650.42	0000	
19.47	3	.....			74.77	2.3	.....		
30.73	3	30.71	0000		5678.18	2.3	.....		
4868.23	3	.....			5689.39	4.5	.....		
4904.03	2.3	.....			5705.97	3	.....		
50.83	2.3	4950.80	0000		22.98	3.4	.....		
57.78	3	?			51.67	4.5	5723.00	0000	
4979.32	2.3	.....			5792.10	4	5792.14	00	
5016.99	2.3	.....			5858.52	4	5858.50	00	
60.07	2.3	5060.11	0000		5888.61	4	5888.65	0000	
80.23	2.3	.....							

A close examination of this table hardly permits a doubt that numerous cases of actual identity occur among these approximately equal wave-lengths. Nevertheless the extraordinary faintness which almost universally characterizes the solar lines here in question renders exceedingly difficult the final decision of the question of coincidence, even with the use of the greatest possible dispersion; while, on the other hand, this faintness gives a sufficient explanation of the fact that these solar lines appear in only the rarest cases on my plates taken hitherto. The beautiful plates which I have recently obtained with the above-mentioned concave grating will, it is hoped, assist in the further solution of this special question.

The great contrast in the intensity of the solar and *Mo* lines, which are to be regarded as belonging to the general solar

spectrum, from what has preceded directly suggests the analogous, though less pronounced, relations which I have previously observed in connection with the spectrum of vanadium. In this metal also only the principal lines of the arc-spectrum are represented in the general solar spectrum, but the lines offer considerably less difficulty in observation on account of a less excessive faintness. Since these vanadium lines have a considerable intensity in Sun-spots according to Young's observations, it would be natural to suspect a similar behavior of the molybdenum lines. To my knowledge nothing is known on this point, however, and the decision of the question will necessitate investigations of the spot spectra of much greater completeness than those which are at present available.

THE VARIABLE STAR 7582 *X CEPHEI*.

By J. A. PARKHURST.

THE variability of this star (R. A. $21^h\ 3^m\ 38^s.5$; Dec. $+82°\ 39'\ 50''.3$ (1900) was noticed by Madame Ceraski in 1898 from an examination of photographs taken at the Moscow Observatory by M. Blajko).[1] Brief notices in regard to the star have been published in various journals, to which reference will be made later, but so far no report approaching completeness has appeared.

INSTRUMENTS.

The visual and photometric observations were made with a 6-inch (157 mm) Brashear reflector and the 12 (305 mm) and 40-inch (101 cm) refractors of the Yerkes Observatory; the photograph of the field reproduced in Plate I was made with the 24-inch reflector, the original negative having a field three inches square, covering 1° 52′. The central part of this field, enlarged four and one-half diameters, appears in the plate.

The photometric observations were made with the equalizing wedge photometer devised by Professor E. C. Pickering and described in this JOURNAL, **13**, 249;[2] the constant of Wedge II there found, **0.130** magnitude, being used. This value of the constant has been confirmed by Mr. Edward S. King at the Harvard College Observatory.[3] In 1902 a different wedge (V) was used, whose constant proved to be somewhat different from Wedge II. A new determination of the constant of Wedge V, as yet unpublished, was made in the same manner as that of Wedge II, the resulting value, **0.110** magnitude, being used in the reductions.

POSITION OF THE VARIABLE.

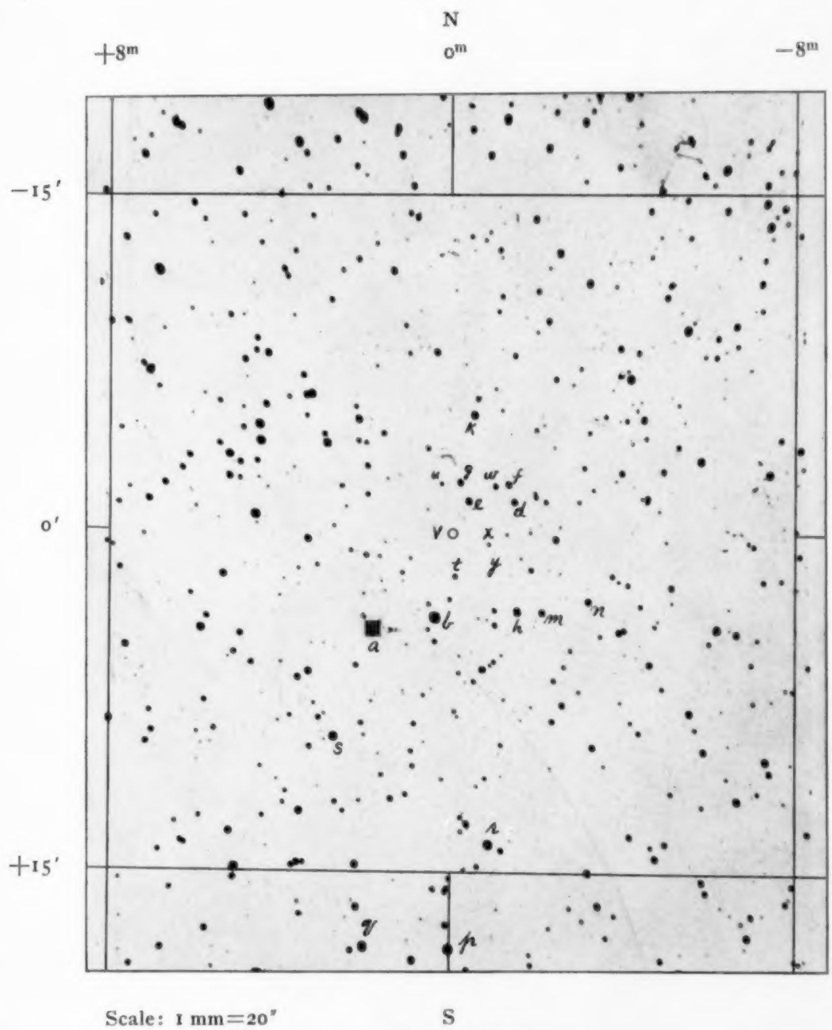
The variable was connected with the comparison stars *a* and *b* on October 11, 1898, with the filar micrometer on the 6-inch

[1] *Astronomische Nachrichten*, **147**, 141.

[2] See sectional drawing, Fig. 1, *loc. cit.*

[3] *H. C. O. Annals*, XLI, 242.

PLATE I.



Scale: 1 mm=20″

7582 *X CEPHEI.*

($21^h\ 3^m\ 38^s.5$; $+82°\ 39'\ 50''$.)

reflector, but, as it was thirteenth magnitude at that time, the position found was not very precise, differing nearly 13″ from the final place. The measures were repeated on October 3, 1899, when the variable had risen to tenth magnitude; with the following results:

b = *Carrington* 3222 - - -	21^h	06^m	$59^s.5$	$+82°$	$25'$	$05''.7$ (1855)
Precession - - -		-02	57.5		$+10$	53.2
b - - - - - - -	21	04	02.0	82	35	58.9 (1900)
$\Delta(bv)$ - - - - - -		-00	23.1		$+03$	51.4
v from b - - - - - -	21	03	38.9	82	39	50.3 (1900)
a = *Carrington* 3225 - -	21	08	22.7	82	24	28.9 (1855)
Precession - - -		-02	55.0		$+10$	57.0
a - - - - - -	21	05	27.7	82	35	25.9 (1900)
$\Delta(av)$ - - - - - -		-01	49.6		$+04$	24.3
v from a - - - - -	21	03	38.1	82	39	50.2 (1900)
v mean - - - - - -	21	03	38.5	82	39	50.3 (1900)
Precession - - -		$+03$	00.7		-10	52.2
v - - - - - - - -	21	06	39.2	82	28	58.1 (1855)

COMPARISON STARS.

When the variable had faded below the limit of the 40-inch refractor I sent a request to Professor Keeler, the late director of the Lick Observatory, to have the field photographed with the Crossley reflector, and he kindly sent me a plate taken July 24, 1900, with an exposure of two hours. The co-ordinates of the comparison stars were measured on this plate by Dr. Caroline E. Furness, with the Repsold measuring machine of Vassar College Observatory, for which courtesy I wish to record in this place my thanks. The results of these measures, with the magnitudes described later, are given in Table I.

The measurement of the light of these comparison stars, one of the principal objects of this investigation, was carried on in the following manner:

Five *Meridian Photometer* stars, lying near the field, were

TABLE I.

Comparison Stars for *7582 X Cephei.*

Letter	Co-ordinates from V		Light		Letter	Co-ordinates from V		Light	
	R. A.	Dec.	Mag.	Steps		R. A.	Dec.	Mag.	Steps
	s	′ ″				s	′ ″		
n	−190	− 3 11	12.68	25.8	*k*	− 30	+ 5 11	12.29	29.0
l	−144	− 0 23	12.79	25.8	*e*	− 23	+ 1 20	12.43	24.4
m	−125	− 3 39	12.0	29.0	*g*	− 10	+ 2 11	12.66	22.9
h	− 91	− 3 35	11.99	29.0	*t*	− 4	− 2 1	14.18	17.8
d	− 87	+ 1 19	12.88	25.8	*p*	+ 1	−18 35	10.2	37.4
f	− 78	+ 2 5	13.23	22.5	*u*	+ 16	+ 2 7	14.88	13.5
w	− 60	+ 2 0	13.98	17.0	*b*	+ 23	− 3 51	9.12	42.3
r	− 51	−13 57	10.08	37.8	*a*	+109	− 4 24	8.24	....
y	− 55	− 0 56	17.0	0.0	*q*	+115	−18 29	10.6	36.5
x	− 50	− 0 36	15.63	7.5	*s*	+160	− 9 11	10.86	32.8

b = *B. D.* + 82° 635 = *Carrington* 3222.
a = *B. D.* + 82 636 = *Carrington* 3225.

TABLE II.

Standard Magnitude Stars from *Harvard Annals*, XXIV.

Letter	*B. D.*		1855		H. C. O.
	No.	Mag.	R. A.	Dec.	Mag.
			h. m.	° ′	
A	+82° 648	8.0	21 30.2	+82 21	7.99
B	+81 737	7.5	21 25.2	+81 54	7.83
C	+81 736	7.9	21 24.7	+81 24	7.47
D	+81 735	7.8	21 23.5	+81 8	7.73
E	+81 742	8.0	21 32.4	+81 6	8.07

selected from *Harvard Annals*, XXIV; their notation, positions, and photometric magnitudes being given in Table II. With these stars as standards, the magnitude of four comparison stars between eighth and eleventh magnitudes, *a*, *b*, *s*, and *r*, were measured with the photometer attached to the 6-inch reflector. From these as standards, four stars between the eleventh and thirteenth magnitudes were measured with the 12-inch; and finally the remaining stars were measured with the 40-inch. That the reader may be able to form an independent estimate of the merits of the methods and instruments for this work, the measures are given in detail in Table III, which is nearly self-explanatory.

There are three main dangers to be guarded against in this work: (1) the lack of close resemblance between the real and artificial stars; (2) variation in the light of the artificial star; and (3) variation in the transparency of the atmosphere, from changing zenith distance or other causes. To insure resemblance in the star images the movable diaphragm close to the lamp was provided with five holes, 0.10, 0.15, 0.20, 0.25, and 0.30 mm in diameter, the choice of the hole used being governed by the telescope and atmospheric conditions of each evening. To eliminate the effect of variation in the light of either star, the series of scale readings was repeated in inverse order (given under "Second" in Table III). An inspection of the scale readings and their means will therefore show any variations, either irregular or progressive, in the light of either star, and the final means will tend to eliminate their effects.

In each division of Table III the stars used as standards are given first, with their magnitudes in bold-faced type. The magnitudes sought for the remaining stars were deduced in the following manner: Means were formed of the scale readings (d_0) and magnitudes (M_0) of all the standards. The difference between the mean scale reading for each star and d ($d-d_0$), was converted into magnitudes by multiplication into the wedge constant, and its value (ΔM) added to M_0.

An idea of the errors in the settings and the maximum effect of irregular changes in the light of the stars can be gained from the average deviations of the separate settings from the means. The values are:

For the 6-inch - - - -	0.06 mag.
For the 12-inch - - - -	0.09 mag.
For the 40-inch - - - -	0.10 mag.

The agreement of the final magnitude results is as follows:

Average Deviation of One Night's Measure from Mean

6-inch - - - -	0.10 mag.	4 stars
12-inch - - - -	0.04 mag.	4 stars
40-inch - - - -	0.08 mag.	7 stars

TABLE III.

Photometric Measures of Comparison Stars.

1900, October 19; 6-inch. Wedge II; seeing very good.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
A	16.5	16.4	17.2	18.1	19.1	18.0	17.55	7.99
B	15.4	15.1	15.8	16.2	16.8	14.7	15.67	7.83
C	12.0	11.0	11.2	12.5	12.6	11.6	11.82	7.47
D	14.2	13.3	14.6	13.4	13.5	12.9	13.65	7.73
E	18.0	17.5	17.2	14.9	16.1	15.0	16.45	8.07
a	17.9	18.8	16.9	17.4	17.4	17.9	17.72	8.24
b	24.0	23.9	22.9	23.2	24.0	23.1	23.52	9.12
s	36.4	36.9	38.4	38.9	37.1	38.8	37.85	10.86
r	34.2	33.4	33.2	32.0	32.0	30.8	32.60	10.08
p	32.4	34.5	33.8				33.57	10.02
q	37.0	35.9	37.2				36.70	10.6
h	44.0	46.3	44.9				45.07	12.0

1900, November 2; 6-inch. Wedge II; Moonlight.

E	16.0	16.0	15.8	16.1	16.8	15.2	15.82	8.07
D	12.7	13.0	12.3	13.9	13.2	14.6	13.29	7.73
C	11.0	11.7	11.4	11.0	11.6	10.4	11.19	7.47
B	15.5	16.2	15.8	15.4	16.1	16.0	15.83	7.83
A	16.7	17.4	17.4	17.9	17.4	17.0	17.30	7.99
a	18.4	18.9	18.2	18.1	18.8	18.2	18.44	8.31
b	25.8	26.3	24.8	25.8	26.0	24.4	25.52	9.23
s	38.3	38.1	38.2	36.8	37.3	36.2	37.49	10.78
r	34.3	32.9	33.9	33.7	32.6	32.0	33.24	10.23

1900, December 12; 6-inch. Wedge II.

E	19.8	20.8	20.8	20.7	20.8	20.0	20.49	8.07
D	18.8	19.4	18.3	16.9	16.8	16.2	17.73	7.73
C	16.5	17.6	17.1	15.7	16.1	15.8	16.45	7.47
B	19.4	20.3	20.0	18.0	18.6	17.0	18.87	7.83
A	21.9	21.0	21.2	20.8	20.1	20.6	20.94	7.99
b	29.9	29.8	29.0	28.8	31.0	29.8	29.72	9.22
a	22.8	22.0	22.4	22.5	20.8	22.1	22.10	8.24
h	51.2	53.8	52.4	54.4	53.5	52.7	53.00	12.2
r	35.2	36.0	36.0	35.1	34.0	34.0	35.05	9.92
s	42.7	44.2	43.0	44.4	43.8	42.2	43.39	11.00

TABLE III.—*Continued.*

RESULTING MAGNITUDES FROM MEASURES WITH 6-INCH.

Star	Oct. 19	Nov. 2	Dec. 12	Mean
a	8.17	8.31	8.24	8.24
b	8.92	9.23	9.22	9.12
s	10.79	10.78	11.00	10.86
r	10.10	10.23	9.92	10.08
p	10.23		.....	10.2
q	10.64		.....	10.6

1900, Sept. 12; 12-inch. Wedge II; moonlight, clear.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
a	12.7	13.2	13.2	13.5	13.1	13.1	13.13	**8.24**
b	20.5	20.0	20.6	21.6	20.9	20.8	20.74	**9.12**
h	45.0	41.5	43.9	40.3	42.0	43.1	42.30	11.98
n	51.1	46.5	48.6	47.4	47.9	46.4	47.98	12.71
l	49.0	51.8	48.8	48.2	49.3	47.0	49.02	12.84
d	47.2	49.6	48.0	48.2	49.9	49.9	48.90	12.84

1900, October 25; 12 inch. Wedge II; seeing good.

a	12.5	12.9	13.7	13.0	13.0	11.9	12.83	**8.24**
b	20.9	21.0	20.9	21.1	20.9	21.0	21.61	**9.12**
s	29.2	30.1	30.0	30.0	30.9	29.9	30.22	**10.86**
d	49.0	48.7	48.3	48.0	47.0	47.5	48.09	12.92
h	41.3	42.3	41.0	41.2	42.1	40.8	41.45	12.00
n	45.0	47.8	47.7	47.4	46.5	44.1	46.42	12.64
l	46.1	46.1	44.2	49.0	47.0	45.0	47.09	12.73
				49.1	49.5	50.8		

RESULTING MAGNITUDES FROM MEASURES WITH 12-INCH.

Star	Sept. 12	Oct. 25	Mean
h	11.98	12.00	11.99
d	12.84	12.92	12.88
l	12.84	12.73	12.79
n	12.71	12.64	12.68

TABLE III.—*Continued.*

1900, September 13; 40-inch. Wedge II; seeing good.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
s	14.2	18.2	15.9	16.9	15.1	14.1	15.74	**10.86**
h	22.4	22.2	22.8	22.2	23.3	22.0	22.49	**11.99**
d	25.8	28.1	26.8	....	....	....	26.90	**12.88**
l	29.8	27.2	29.2	28.5	27.0	30.0	28.62	**12.79**
t	38.8	40.8	38.8	37.0	37.4	36.9	38.29	14.06
x	49.3	49.3	47.1	49.3	49.4	50.0	49.07	15.47
f	31.1	29.9	32.1	....	....	....	31.10	13.13
w	36.2	37.2	36.2	....	....	....	36.53	13.82
e	26.1	26.2	26.0	25.9	24.0	27.5	25.95	12.46
g	26.5	29.5	27.2				27.73	12.69
u	43.8	43.3	44.4				43.83	14.78
k	26.7	22.8	24.6				24.70	12.29

1902, January 7; 40-inch. Wedge V; seeing good.

d	21.9	25.7	24.7	24.8	23.6	24.7	25.4	24.1	24.42	**12.88**
l	29.2	27.9	29.9	28.2	22.5	23.5	21.8	21.8	25.60	**12.79**
n	22.2	24.0	21.7	23.2	20.4	20.4	18.8	19.3	21.18	**12.68**
e	17.0	20.3	20.4	20.9	22.0	19.7	21.8	20.1	20.15	12.39
x	49.8	51.7	48.8	50.7	51.1	52.8	52.0	50.6	50.94	15.77
g	20.0	23.8	21.8	23.2	23.2	22.5	21.8	21.8	22.27	12.62
w	38.1	39.2	38.6	38.2	32.9	33.1	33.8	33.3	35.91	14.12
f	28.7	29.5	30.6	28.9	28.2	28.8	28.0	26.2	28.62	13.32
t	33.2	35.0	33.7	36.0	37.2	37.2	37.4	37.8	35.94	14.12
u	42.9	44.3	42.7	43.7	43.5	43.4	44.2	43.8	43.57	14.96

RESULTING MAGNITUDES FROM FIRST TWO SETS WITH 40-INCH.

Star	Sept. 13	Jan. 7	Mean
e	12.46	12.39	12.43
k	12.29	.....	12.29
g	12.69	12.62	12.66
f	13.13	13.32	13.23

TABLE III.—*Continued.*

1900, September 6; 40-inch — Wedge II; bright Moon.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
d	22.0	21.5	23.0	29.4	27.8	25.1	24.80	**12.88**
l	28.2	29.5	29.2	33.0	30.5	29.9	30.05	**12.79**
e	25.4	24.8	27.0	25.4	25.0	23.6	26.20	**12.43**
g	27.8	28.0	28.1	26.3	30.3	27.0	27.92	**12.66**
f	30.6	31.8	32.8	30.0	32.7	33.0	31.82	**13.23**
w	38.0	35.9	35.3	36.8	39.0	39.0	37.34	13.99
t	38.7	40.8	40.6	....	....	....	40.05	13.35
u	41.9	44.2	43.1	46.8	45.0	45.5	44.42	14.91

1900, May 8; 40-inch. — Wedge II; seeing poor.

e	28.8	31.2	28.2	27.2	30.4	27.7	28.5	31.8	29.23	**12.43**
g	27.8	27.6	31.2	28.5	32.2	31.8	33.2	26.5	29.86	**12.66**
t	42.2	41.8							42.0	14.2
u	48.9	48.9							48.9	15.1

RESULTING MAGNITUDES FOR *t*, *u*, and *w* WITH 40-INCH.

Star	Sept. 13	Jan. 7	Sept. 6	May 8	Mean
t	14.06	14.12	14.35	(14.2)	14.18
u	14.78	14.96	14.91	(15.1)	14.88
w	13.82	14.12	13.99	......	13.98

1902, May 29; 40-inch. — Wedge V; seeing fairly good.

Star	Scale Readings								Mean Scale Reading	Mag.
	First				Second					
t	39.0	39.2	38.4	39.8	39.5	39.4	40.1	39.0	39.30	**14.18**
u	44.7	45.4	45.6	45.8	45.0	45.0	46.0	45.5	45.38	**14.88**
x	52.4	52.6	52.5	54.0	52.2	50.5	52.0	53.5	52.47	15.66

RESULTS FOR ALL MEASURES OF *x* WITH 40 INCH.

Star	Sept 13	Jan. 7	May 29	Mean
x	15.47	15.77	15.66	15.63

MAGNITUDE CURVE.

The visual observations of the variable were made by Argelander's method, and a light-scale formed from the resulting intervals between the comparison stars, expressed in steps. Fig. 1 shows the correspondence between the light of the stars expressed in steps and the magnitude found by the photometer.

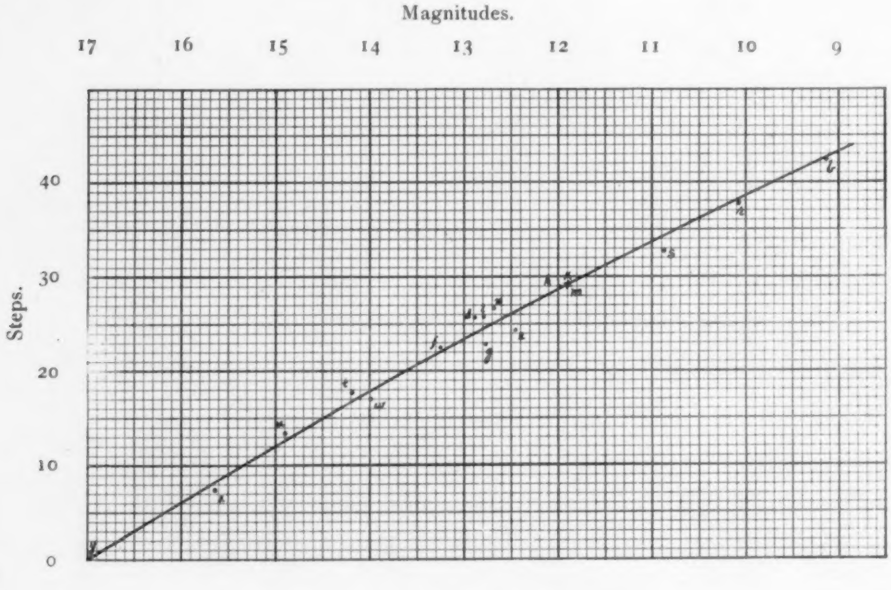


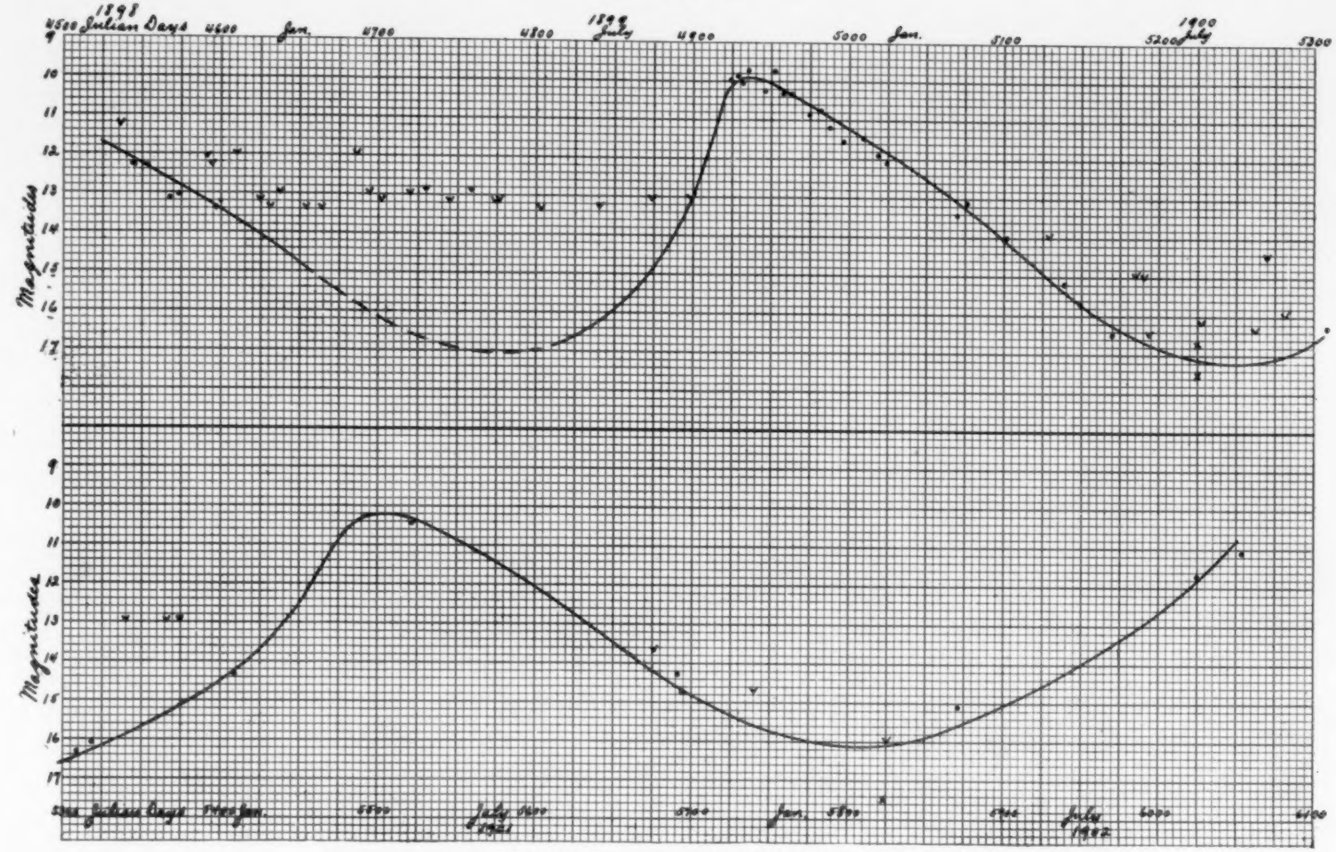
FIG. 1.—Magnitude Curve of *X Cephei.*

It is platted with the photometric magnitudes as abscissæ and the step values in the light-scale as ordinates. The average distance between the platted points and the smooth curve drawn through them is about 0.10 magnitude. This curve serves to fix the magnitudes of the three stars, *y*, *k*, and *m*, which were not measured with the photometer, and also to translate the brightness of the variable, expressed in steps in the comparisons, into magnitudes.

VISUAL OBSERVATIONS OF THE VARIABLE.

These observations are given in detail in Table IV, the headings of the columns leaving little explanation necessary. In the

PLATE II.



Light Curve of *X Cephei*.

column headed "Details in Steps" the light of the variable is deduced from the position of each comparison star in the light-scale as given in the last column of Table I. When the variable was invisible, the brightness of the faintest comparison star which could be seen is given, preceded by the inequality sign, $<$, which should be read "fainter than," instead of in its usual significance, "numerically less than," the quantity following. In the comparisons, v stands for the variable.

THE LIGHT CURVE OF THE VARIABLE.

Plate II gives a graphic representation of the star's variations, platted with time as abscissæ and magnitudes as ordinates. The visual magnitudes are shown in dots and the photographic magnitudes from the two plates, July 24, 1900, and March 13, 1902, in small crosses. These two latter are not directly comparable with the visual results, a difference of one-half magnitude or more being indicated. On dates when the variable could not be seen the limiting magnitude visible is indicated by the sign v. The curve drawn through the platted points shows the maximum magnitude to be 9 or 10. This agrees sufficiently well with the visual and photographic observations of Ceraski and Blajko,[1] who found it between ninth and tenth magnitude in May and June, 1898. The magnitude at minimum is not far from 17, being below the limit of the 40-inch with a low power.

The Julian dates of the two maxima are about 241 4935 and 241 5500; of the minima, 241 5250 and 241 5810; the intervals being 565 and 560 days, respectively. The sudden rise to maximum in 1899 is not entirely confirmed by the other parts of the curve, but the observations seem definite on that point.

LIMITING MAGNITUDES.

A point of interest in regard to the photometry of faint stars is touched by the limiting magnitudes of the different apertures

[1] *A. N.*, 153, 297.

TABLE IV.

7582 X Cephei

Comparisons of the Variable by Argelander's Method.

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1898		C. S. T.	G. M. T.			
1	Sept.	3	9	4536.6	80	6	*a* and *b* seen, nothing else near
2		6	9	4539.6	150	6	*v* 1-2 *d*, *d* 2 *e*, *e* 1 *f*, *e* 1 *g*
3		7	9	4540.6		6	No change
4		20	8	4553.6	150	6	*h* 2 *v*. *v* limit, is this *v*?
5	Oct.	5	8	4568.6	150	6	*e* 2 *v*, *e* 1 *g*, *e d*, *d* 3 *f*
6		11	8	4574.6	80	6	*g* 1 *v*, *e* 1-2 *v*, *d* 2 *v*, *v* 2 *f*
7		28	6	4591.5	150	6	*v*, *d*, *f*, *e* and *g* not seen
8		31	6	4594.5	80	6	*v*, *d*, *f*, *e* and *g* not held
9	Nov.	2	7	4596.5	200	6	*e* 3 *v*, *g* 1-2 *v*, *f* 1-2 *v*
10		6	7	4600.5	80	6	*e*, *g* and *d* seen, *f and v* glimpsed
11		16	7	4610.5	200	6	Nothing seen near *v*
12		30	6	4624.5	150	6	*e*, *d*, *f* and *g* seen, *v* not seen
13	Dec.	7	7	4631.5	200	6	*v* not seen
14		13	6	4637.5	150	6	*v* not seen
15		30	6	4654.5	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
	1899						
16	Jan.	8	6	4663.5	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
17	Feb.	1	6	4686.5		6	*d*, *e*, *f*, *g* and *v* not seen
18		7	7	4692.5		6	*d*, *e* and *g* seen, *v* not seen
19		15	18	4702.0	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
20	Mar.	6	7	4720.5	150	6	*d*, *e*, *f* and *g* seen, *v* not seen
21		15	8	4729.5		6	*e* seen, *v* not seen
22		31	8	4745.5	150	6	*d*, *e*, *f* and *g* seen, *v* suspected
23	Apr.	12	8	4757.6	150	6	*e* and *d* seen, *v* not seen
24		28	9	4773.6	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
25	May	1	9	4775.6	200	6	*d*, *e*, *f* and *g* well seen, *v* not seen
26		29	9	4804.6	200	6	*d*, *e*, *f* and *g* well seen, *v* not seen
27	July	6	10	4842.7	150	6	*d*, *e*, *f* and *g* well seen, *v* not seen
28	Aug.	5	9	4872.6	150	6	*d*, *e*, *f* and *g* seen, *v* not seen
29		30	9	4897.6	150	6	*d*, *e* and *g* seen, *v* not seen
30	Sept.	26	7	4924.5	150	6	*b* 4 *v*, *v* 0-1 *r*, *v* 1-2 *p*
31		30	7	4928.5	150	6	*b* 2-3 *v*, *v* 1-2 *r*, *v* 1 *p*
32	Oct.	3	8	4931.6	80	6	*b* 3-4 *v*, *v r*
33		6	7	4934.5	40	6	*b* 2-3 *v*, *v* 2 *r*
					150	6	*b* 3-4 *v*, *v* 2-3 *r*
34		18	7	4946.5	80	6	*b* 4 *v*, *r* 1 *v*, *v p*, *v* 1 *q*
35		24	7	4952.5	40	6	*b* 3-4 *v*, *v* 2-3 *r*, *v* 2-3 *p*
36		29	7	4957.5	150	6	*b* 4-5 *v*, *r* 2-3 *v*
					40	6	*b* 4 *v*, *r* 1 *v*, *v* 4 *s*
37	Nov.	4	7	4963.5	150	6	*b* 5 *v*, *v* 3 *s*
					40	6	*b* 4-5 *v*, *v r*, *v* 3 *s*
38		15	7	4974.5	150	6	*b* 10 *v* ±, *r* 2 *v*, *v* 2 *s*
39		22	7	4981.5	40	6	*b* 6-8 *v*, *r* 3 *v*, *v* 2 *s*
40		28	7	4987.5	150	6	*r* 4 *v*, *s* 1 *v*, *v* 4 *h*
41	Dec.	6	7	4995.5	150	6	*s* 2 *v*, *v* 2 *h*
42		19	6	5008.5	150	6	*s v*, *v* 1-2 *h*

TABLE IV.

7582 X Cephei

Reductions of Observations.

	Details in Steps	Means		Seeing	Remarks
		Steps	Mag.		
1		<27	<11.3	moon	limit 10<*b*
2	27.3	27.3	12.27	fair	limit *g*
3			12.3±		
4	27.0	27.0	12.30	fair	limit *v*, is it *v*?
5	22.4	22.4	13.16	good	
6	21.9, 22.9, 23.8, 24.5	23.2	13.01	good	limit *f*
7		<28	<12.1	full moon	limit 1<*h*
8		<27	<12.3	fair	limit 2<*h*
9	21.4, 21.4, 21.0	21.2	13.40	good	
10		22±	13.2±	good	
11		<29	<12	fair	limit *h*
12		<22	<13.2	good	limit *f* =3 <*e*
13		<21	<13.4	good	limit 1 < *f*
14		<23	<13.0	fair	limit *g*
15		<21	<13.4	good	limit 1 < *f*
16		<21	<13.4	good	limit 1 < *f*
17		<29	<12	poor	limit *h*
18		<23	<13.0		
19		<22	<13.2	good	
20		<23	<13.0		
21		<24	<12.9	clouds	
22		<22	<13.2	good	
23		<24	<12.9	good	
24		<22	<13.2	good	
25		<22	<13.2	good	
26		<22	<13.4	good	
27		<22	<13.3	good	
28		<23	<13.1		
29		<23	<13.1	fair	
30	38.3, 38.3, 38.9	38.5	10.00	good	
31	39.8, 39.3, 38.4	39.1	9.88	good	
32	38.8, 37.8	38.3	10.05	good	
33	39.8, 39.8 38.8, 40.3	39.9	9.74	fair to good	
34	38.3, 36.8, 37.4, 37.5	37.5	10.22	moon, fair	
35	38.8, 40.3, 39.9	39.7	9.76	good	
36	37.8, 34.8 38.3, 36.8, 36.8	36.9	10.33	good	
37	37.3, 35.8 37.8, 37.8, 35.8	36.9	10.33	good	
38	32.3, 35.8, 34.8	34.2	10.87	moon, fair	
39	35.3, 34.8, 34.8	34.9	10.78	fair to good	
40	33.8, 31.8, 33.0	32.8	11.20	fair to good	
41	30.8, 31.0	30.9	11.57	fair	
42	32.8, 30.5	31.6	11.44	fair	

TABLE IV—*Continued.*

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1899		C. S. T.	G. M. T.			
43		29	7	5018.5	150	6	*s* 3 *v*, *h* 1 *v*, *v* 4 *l*
	1900						
44	Jan.	4	7	5024.5	150	6	*s* 4-5 *v*, *h* 2 *v*, *v* 3 *l*
45		21	6	5041.5	350	40	
46	Feb.	18	14	5069.8	350	40	*e* 8 *v*, *v* 8 *t*, *v* 2 *e*
47		25	9	5076.6	275	12	*e* 1 *v*, *v f*, *v* 3 *t*
48	Mar.	21	8	5100.6	275	12	*t* 1 *v*, *v* 1 *w*
49		22	10	5101.7	350	40	*v t*, *v* 1 *w*
50	Apr.	18	13	5128.8	350	40	*v* not seen, *t* glimpsed
51		27	12	5137.8	237	40	*w* 6 *v*, *t* 6 *v*, *u* 3 *v*, *v* 3 *x*
					350	40	*u* 5 *v*
52	May	8	14	5148.8	460	40	*u* 6 *v*, *x* 0-1 *v*
					460	40	*u* 5 *v*, *x* 1 *v*, *w* 8 *v*, repeated
53		29	13	5169.8	237	40	*x* 4-5 *v*
54	June	13	12	5184.8	350	40	*v*, *u* or *x* not seen, *w* seen
55		19	14	5190.8	350	40	*v* and *x* not held
56		21	12	5193.0	237	40	*v* not seen
57	July	24		5225			
58		25	15	5226.9	460	40	*v* not seen
59	Aug.	29	15	5261.9	460	40	*v* not seen
60	Sept.	6	9	5269.6	237	40	*v* or *x* not seen
61		13	8	5276.6	237	40	*v* not seen
62	Oct.	16	9	5309.6	460	40	*x* 3 4 *v*, *v* 4 *y*
63		25	11	5318.7	460	40	*x* 2 *v*
64	Nov.	15	6	5339.5	150	6	*v* not seen
65	Dec.	11	6	5365.5	150	6	*v* not seen
66		19	6	5373.5	150	6	*v* not seen
	1901						
67	Jan.	24	7	5409.5	350	40	*t* 2 *v*, *v* 2 *u*
68	May	16	8	5521.6	80	12	*b* 5 *v*, *r* 2 *v*, *v* 4 *s*
69	Oct.	18	8	5676.6	80	12	*v* not seen
70	Nov.	1	8	5690.6	275	12	*t* 2 *v*, *v* 1 *w*, *v* 1 *u*
71		4	9	5693.6	460	40	*t* 4 *v*, *w* 4 *v*, *v u*
72	Dec.	21	7	5740.5	350	40	*v* not seen
	1902						
73	Mar.	13	11	5827.0			24-inch reflector, $v\ \frac{1}{2}^{m} \pm < y$
74		15	14	5824.9	460	40	*v* or *y* not seen
75	May	2		5872	237	40	*t* 6 *v*, *u* 4 *v*, *v* 4 *x*
76		29	9	5899.6	237	40	photometer
77	Oct.	1	9	6024.6	237	40	*s* 2-3 *v*, *b* 10 *v*, *v* 10 *h* ±
78		31	9	6053.6	237	40	*s v* ±

TABLE IV — *Continued*

	Details in Steps	Means		Seeing	Remarks
		Steps	Mag.		
43	29.8, 28.0, 29.8	29.2	11.89	good	
44	28.3, 27.0, 28.8	28.0	12.11	fair	
45				good	Nothing as bright as 16 *m* within 1'.5 of *v*
46	16.4, 25.8, 20.9	21.0	13.42	fair, moon	
47	23.4, 22.5, 20.8	22.8	13.11	good	limit *t*
48	16.8, 18.0	17.4	14.05	good	limit *w*
49	17.8, 18.0	17.9	14.00	good	
50		<18	<14.0	moon, haze	
51	11.1, 11.8, 10.5, 10.5	10.9	15.20	good	limit, 2–3 < *x*
52	8.5, 7.5, 7.0 7.5, 7.0, 9.0 8.5, 6.5, 9.0	7.8	15.71	fair	limit *v*
53	3.0	3.0	16.50	fair	limit *v*
54			<15	moon	limit 1 < *t*
55			<15	fair	limit 1 < *t*
56		< 3.0	<16.5		limit 4–5 < *x*
57			<16.5		Crossley plate
58		< 4.5	<16.2	good	limit 3 < *x*
59		< 3.5	<16.4	good	limit 4 < *x*
60		<13.5	<14.7	moon	limit *u*
61			<16		limit ½ *m* < *x*
62	4.0, 4.0	4.0	16.33		limit *y*
63	5.5	5.5	16.09	thro' clouds	
64		<22	<13	good	
65		<22	<13	good	
66		<23	<13	good	
67	15.8, 15.5	15.6	14.39	fair	
68	37.3, 35.8, 36.8	36.6	10.42	good	
69		<19.4	<13.7	fair	limit 3–4 < *g*
70	15.8, 18.0, 14.5	16.1	14.30	good	limit 1 < *w*
71	13.8, 13.0, 13.5	13.4	14.76	poor	limit 1 < *u*
72		<13.5	<14.7	fair, moon	limit *u*
73			17½ ±	good	
74		< 5	<16	unsteady	limit 2–3 < *x*
75	11.8, 9.5, 11.5	11.3	15.10	good	
76			15.21	fairly good	
77	30.3, 32.3, 39.0	30	11.7 ±	poor	
78		33	11.1 ±	poor	

used in this work. If we assume 13.0 magnitude as the limit of 6 inches aperture, we have the following table:

Aperture		6	12	40
Magnifying power		200	275	460
		M	M	M
Limit	Observed	13.2	14.2	17.0
	Calculated	13.0	14.5	17.

The visibility of stars fainter than the theoretical limit of a 6-inch reflector seems due to the use of a (proportionally) high power, while the lack in the 12-inch seems due to the comparatively low power.

I wish in this place to acknowledge my indebtedness for assistance in the work to Miss Kate Bloodgood, who has formed the light-scale and reduced the visual observations; to Mr. Ferdinand Ellerman for preparing the photograph of the field for the engraver, and to Mr. Frank Sullivan, who recorded the photometric measures with the 40-inch.

YERKES OBSERVATORY,
December 1902.

THE PROPER MOTIONS OF THE STARS.

By GAVIN J. BURNS.

OF all methods of investigating the problem of the constitution of the stellar universe, none is more promising than the study of the proper motions of the stars. Some of the inferences that may be drawn from what is already known of the proper motions of the stars will be considered in this paper.

The material for this investigation has been obtained from the catalogue of the proper motions of 2,641 stars by M. J. Bossert, published in the *Annales de l'observatoire de Paris* in 1896. This catalogue is confined to proper motions of over $0^{s}.01$ in R.A. or $0''.1$ in declination which have been deduced from observations extending over at least fifty years. The following table gives a summary of the stars contained in this catalogue, arranged according to magnitudes. The numbers in the first column are proper motions measured on the arc of a great circle:

PROPER MOTION	MAGNITUDE									TOTAL
	1 to 1.9	2 to 2.9	3 to 3.9	4 to 4.9	5 to 5.9	6 to 6.9	7 to 7.9	8 to 8.9	9 and over	
Sec. per annum	No.	No.	No.	No.	No.	No.	No.	No.	No.	
0.1 to 0.2	4	10	33	68	128	218	265	236	70	1,032
0.2 to 0.4	4	12	23	50	91	213	271	292	67	1,023
0.4 to 0.8	3	7	9	22	24	64	88	113	24	354
Over 0.8	4	0	8	11	17	30	30	29	14	143
Total	15	29	73	151	260	525	654	670	175	2,552

On examining the above table it will be found that little connection can be traced between the magnitude of a star and its proper motion. The first two lines, for instance, consist of a nearly equal number of stars distributed throughout the various magnitudes in nearly the same proportion. The numbers

also in columns 6 to 9 show much the same distribution of proper motions. Thus we find that

The average magnitude of a star having a p. m. of $0''.1$ to $0''.2$ is 7.0.
The average magnitude of a star having a p. m. of $0''.2$ to $0''.4$ is 7.1.
The average magnitude of a star having a p. m. of $0''.4$ to $0''.8$ is 7.2.
The average magnitude of a star having a p. m. of over $0''.8$ is 6.9.

I believe it is now generally admitted by astronomers that the proper motion of the stars is, on an average, proportional to their parallax. It follows that a very large proportion of the fainter stars out of the 2,552 above enumerated must be at nearly the same distance as the brighter stars. Each number, in fact, in any horizontal line represents a group of stars at nearly the same average distance from us. Now, if we leave out of consideration stars of the ninth magnitude and over, it will be seen that the numbers increase from left to right with only three exceptions. Hence, if we agree to measure the "size" of a star by the quantity of light it emits, it follows that, so far as the stars actually enumerated are concerned, they become more numerous as they decrease in "size." The apparent exception in the case of stars of the ninth magnitude is doubtless due to lack of material for calculating the proper motion of the fainter stars.

The question here arises as to how far the 2,552 stars under consideration may be taken as representative of the stars in general. Now, it is much more likely that, for any given proper motion and for any two given magnitudes, the number of stars of the fainter magnitude will hereafter be augmented by further observation and research than the number of stars of the brighter magnitude. Taking this into consideration, it appears very probable that the stars do, in fact, increase in number as they decrease in size. Doubtless, there is some particular "size" of star which is more numerous than any other, larger or smaller, but there are no data for determining what it is.

Let us next see what light may be gained on the question of the distribution of stars in space.

The following list shows the number of stars under the seventh magnitude:

Under magnitude 2.00 there are 40 stars.
From 2.00 to 2.99 there are 99 stars $= 2.5 \times 40$.
From 3.00 to 3.99 there are 317 stars $= 3.2 \times 99$.
From 4.00 to 4.99 there are 1,020 stars $= 3.2 \times 317$.
From 5.00 to 5.99 there are 2,865 stars $= 2.8 \times 1{,}010$.
From 6.00 to 6.99 there are 9,554 stars $= 3.4 \times 2{,}865$.

The above numbers, except the last, have been obtained by actual enumeration from the Harvard *Photometries*. The 9,554 stars is an estimate made by Mr. J. E. Gore from the numbers in the Harvard *Photometric Durchmusterung*. Professor Newcomb calls the ratio of the numbers of stars of two successive magnitudes (given above) the "star-ratio." It will be noted that the star-ratio to the seventh magnitude has an average value of 3.

Now, suppose that at the distance x_1 there are n stars of such a size as to appear as stars of the first magnitude; that at the distance x_2 a star of the same size appears as a star of the *second* magnitude, and so on. Suppose, further, that the number of stars at the distance x_1 of the second magnitude is $n\,m_2$, of the third magnitude $n\,m_3$, etc. Then, on the hypothesis that stars of all sizes are distributed in the ratio $1, r, r^2, r^3 \ldots .$ at the distances $x_1, x_2, x_3, x_4 \ldots .$, we find that, if stars nearer than x_1 are neglected,

The number of stars of first magnitude $= n$
The number of stars of second magnitude $= nm_2 + nr$
The number of stars of third magnitude $= nm_3 + nm_2r + nr^2$
The number of stars of fourth magnitude $= nm_4 + nm_3r + nm_2r^2 + nr^3$.

Consequently, if R_1, R_2, R_3 are the star ratios,

$$R_1 = m_2 + r$$

$$R_2 = \frac{m_3}{m_2 + r} + r$$

$$R_3 = \frac{m_4}{m_3 + m_2r + r^2} + r \ .$$

We see from the above formulæ that r is always less than the star-ratio, unless $m = 0$, in which case $R = r$. If all the m's be equal, the star-ratio will constantly diminish for higher magnitude. If, as appears to be the case in reality, the m's increase with the magnitude, r might be much less than the star-ratio. Now, it can be readily shown that for a uniform distribution of

the stars in space $r = 4$. But $R = 3$. Consequently $r < 3$, and probably r is considerably less than 3. This implies that the stars thin out as their distance from us increases. It is true that this reasoning is not demonstrative, for we have assumed r to be constant. Nevertheless it is difficult to avoid the conclusion that with a star-ratio of 3 the stars do in reality thin out as they recede from our system.

The truth of this may be shown in another way. If the stars were all moving at the same average velocity at right angles to the line of sight, the number of stars having angular proper motions lying between a and $2a$ would be eight times the number of those having proper motions between $2a$ and $4a$, upon the hypothesis of uniform distribution. A glance at the table already given will show how greatly the smaller proper motions fall short of the number we should expect. It is true that this defect of small proper motions may be largely due to insufficiency of data, as large proper motions are much more likely to be discovered than small ones. If this were the only cause, we should expect that, as our knowledge extended, the known number of small proper motions would show a relative increase. But this does not seem to be the case. The following table gives the number of stars in Dunkin's list[1] published in 1863 and Bossert's catalogue published in 1896:

Proper Motion in Seconds per Annum.	Dunkin		Bossert	
	No. of stars	Per cent.	No. of stars	Per cent.
0.1 to 0.2	222	60	1,032	40
0.2 to 0.4	99	27	1,023	40
0.4 to 0.8	36	10	354	14
Over 0.8	12	3	143	6
Total	369	100	2,552	100

It will be remembered that Bossert's catalogue omits proper motions under 0.^{s}01 in R. A. Consequently it is deficient in proper motions between 0.″1 and 0.″2 of arc. When allowance is made for this deficiency, the two lists will be found to contain nearly the same proportion of proper motions of different mag-

[1] *Memoirs of R. A. S.*, 32.

nitudes, except for those over 0".8, in which there is a decided increase. This is exactly the opposite of what might have been anticipated. The inference is that there is in reality a much greater number of stars with large proper motions than there would be if they were uniformly distributed throughout the stellar universe.

Upon the whole we appear to be justified in coming to the following conclusions:

1. The stars increase in number as they decrease in size. In other words, smaller stars are more numerous than larger ones.

2. The stars thin out as their distance from the solar system increases.

Another remarkable fact connected with the distribution of proper motions is that double stars have frequently large proper motions. The following comparison illustrates this:

Average p. m. of 778 stars from first to fifth magnitude (average magnitude = 4) contained in Dunkin's list, 0".15.

Average p. m. of 54 double stars from first to seventh magnitude (average magnitude = 4.5) from Struve's catalogue, 0".37.

The following table exemplifies the peculiarity still further:

Magnitude	Average p. m. according to Auwers	Average p. m. according to Kapteyn	Average p. m. of southern double stars (Innes)		Average p. m. of Burnham's double stars	
1	0".263		(7)	0".71	(3)	0".07
2	.137	0".196	(11)	.09	(6)	.05
3	.096		(26)	.18	(12)	.18
4	.075	.147	(44)	.22	(23)	.07
5	.063	.101	(69)	.18	(58)	.08
6	.055	.079	(39)	.12	(71)	.15
7	.049	.066	(24)	.23	(15)	.12
8	.045	....	(12)	.37	(18)	.21

The numbers in brackets denote the number averaged. An allowance of 0.01 has been made for each star described as having a "small" proper motion in Innes' catalogue. It will be noted that it is the fainter stars that have the proper motions above the average.

This apparent large proper motion of double stars is worthy of more attention than seems to have been given to it.

WEYMOUTH, ENGLAND,
December 1, 1902.

THE ORBIT OF THE SPECTROSCOPIC BINARY η *ORIONIS.*

By WALTER S. ADAMS.

THE variation in the radial velocity of η *Orionis* was discovered at the Yerkes Observatory in December of 1901, and announced at the Washington meeting of the Astronomical and Astrophysical Society during that month. The star has been on the regular observing list since that time, and sufficient material has been secured to make an accurate determination of its orbit possible.

The star belongs to the class of binaries in which one component is relatively dark, as no certain evidence of superposed spectra has been found on any of the photographs. The spectrum is that of the *Orion* type, but contains, in addition to the regular helium and hydrogen lines, three silicon lines, and a number of lines due to oxygen and nitrogen which have proved of great value in determining the star's velocity. In general the lines may be said to be slightly better for purposes of measurement than in the case of the average star of this type, though still rather ill-defined and diffuse. At some points in the star's orbit the change in its radial velocity is so rapid as to amount to several kilometers in the course of the exposure required to photograph the spectrum, and this, no doubt, influences to a considerable degree the character of the lines upon the plates taken at such times.

The number of lines measured upon the different plates varies considerably, but in the great majority of cases is from eight to ten. A careful examination of the cases in which more or less lines have been measured has led to the conclusion that any attempt to make a distinction in the weight given to a plate according to the number of its lines would be fully as liable to introduce error as to eliminate it, and accordingly unit weight has been assigned in each case. So far as the individual lines are concerned, however, the method has been adopted of assign-

ing a weight to each line at the time of measurement, representing in the judgment of the observer the accuracy with which the line is measured. As the weights are assigned previous to any knowledge of the velocity values given by the different lines, this procedure is free from any tendency to be influenced by these values in giving the weights.

To illustrate the amount of range shown by the different lines, the results of the measures of two of the plates are given in full below. These plates are taken at random, one from among the earlier observations, and one from among the observations of the past few months, and are of an average quality.

B 269 1902, JANUARY 9, G. M. T. $13^h\ 38^m$			B 448 1902, NOVEMBER 6, G. M. T. $18^h\ 28^m$		
Line	Velocity	Weight	Line	Velocity	Weight
4340.634	+188.9km	2	4345.677	−50.7km	2
4349.541	187.8	1	4367.012	45.5	2
4388.100	199.3	2	4415.076	57.4	3
4415.076	190.0	2	4417.121	52.0	3
4417.121	191.4	2	4471.676	54.2	3
4471.676	195.1	2	4481.400	55.6	2
4552.750	198.4	3	4552.750	57.6	2
4574.900	191.6	1	4567.950	49.0	2
4591.066	192.2	1	4574.900	52.2	2
4649.250	188.8	2	4591.066	56.6	2
4713.308	189.3	3	4596.291	55.3	2
Weighted Mean	+192.4		Weighted Mean	−53.4	
Reduction to Sun	−13.4		Reduction to Sun	+15.3	
Radial Velocity	+179.0 km		Radial Velocity	−38.1 km	

The following table contains the series numbers, dates, and results of measurement of the twenty-eight plates which have been obtained, and which are used in the determination of the orbit. The range of velocity is very great, amounting to over 285 kilometers, and is the largest which has hitherto been found among binaries of this class.

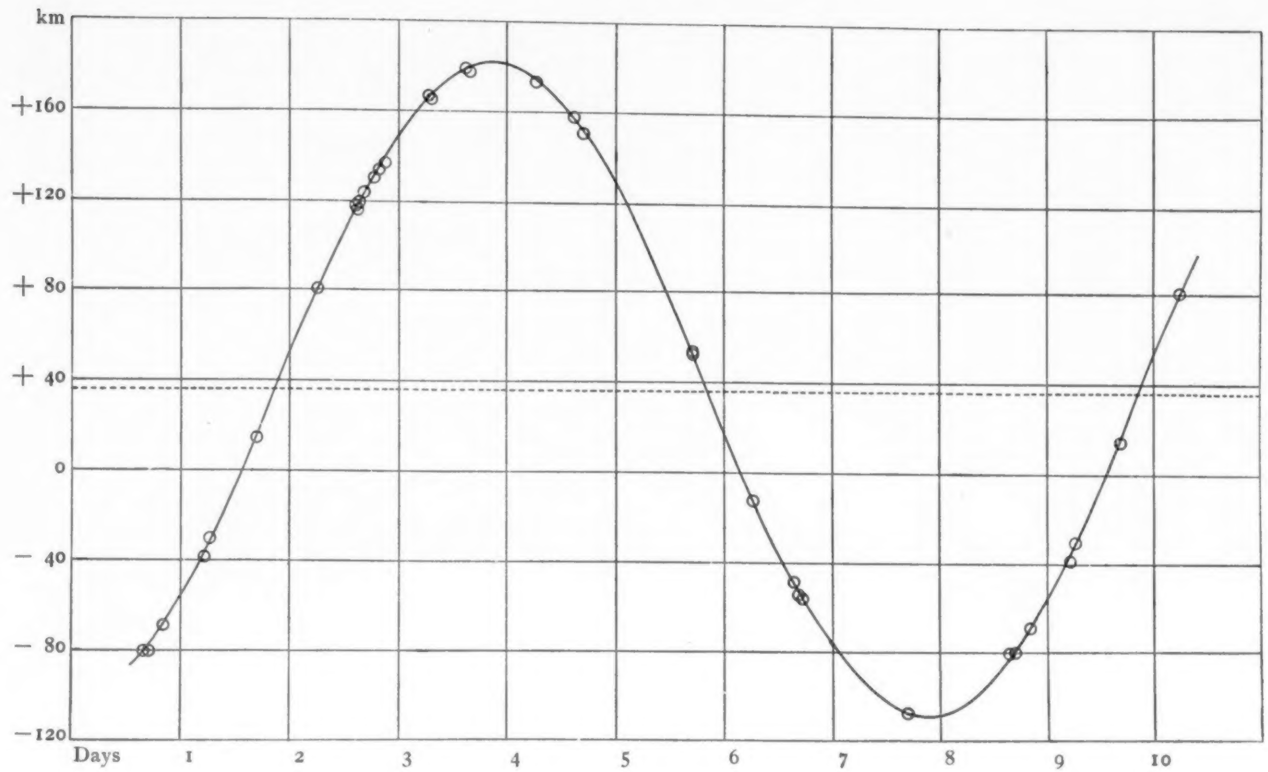
The spectrograms were all taken by the writer, with the exception of the first two and one of the last, which were obtained by Professor Frost.

Plate No.	Date G. M. T.	Velocity	O.–C.
		km.	km.
B 251	1901, November 27.846	– 68.9	–0.6
A 294	December 6.691	+ 14.6	–1.8
B 256	December 18.703	+ 54.4	+1.9
A 296	December 19.676	– 53.7	–0.5
B 260	December 31.585	+115.9	–1.0
B 265	December 31.835	+136.6	–2.4
A 299	1902, January 4.691	– 55.1	+1.2
A 305	January 8.722	+130.0	0.0
B 269	January 9.568	+179.0	+0.1
B 274	January 16.547	+117.6	+2.2
B 279	January 16.762	+133.1	–1.6
A 309	January 22.590	– 80.5	+0.6
B 283	January 24.569	+119.8	+1.3
A 310	February 10.565	+177.8	–1.9
A 315	February 11.525	+158.5	+0.1
A 316	February 12.629	+ 53.1	+1.1
A 321	February 19.598	+151.0	–2.0
A 326	February 21.549	– 47.9	+1.3
B 296	March 13.564	+123.9	+0.2
A 339	March 27.563	– 80.0	–2.6
B 309	April 3.542	–106.5	0.0
B 405	September 3.908	– 29.5	+0.4
A 372	September 4.892	+ 80.5	+1.6
B 411	September 13.901	+166.6	+0.8
B 416	October 8.860	+173.6	–1.9
B 426	October 15.888	+165.0	–2.2
B 448	November 6.769	– 38.1	–2.6
B 460	November 19.822	– 11.8	0.0

All of the above measures were made by Adams with the exception of those of B 269 and B 309, which are the means of measures by Frost and Adams. Plate B 279 was measured twice, and the value given is the mean of the two determinations.

The method used in the derivation of the orbit was that of Lehmann-Filhés, although the very low value of the eccentricity would' no doubt, have made some of the methods using developments in power series equally applicable. The observations were plotted on millimeter paper with a period of $U = 7.9896$ days, and a smooth curve was drawn through them subject to the condition that, after an abscissa had been constructed, making the areas above and below it equal, the adjacent areas included between the maximum and minimum ordinates, the abscissa, and the curve should be equal. The areas in question were adjusted by means of a planimeter, and the following quan-

PLATE III.



Light Curve of η *Orionis.*

tities were obtained as a basis for the computation of the elements:

Velocity of system $V = +35.5$ km.
$A = 146.5$ km; $B = 143.0$ km; $z_1 = +1.35$; $z_2 = -1.38$.

The unit in the cases of z_1 and z_2 is arbitrary, as their ratio alone is required. The notation is that of Lehmann-Filhés.

The elements derived from these quantities are as follows:

$$u_1 = 90^\circ\ 41'.6$$
$$\omega = 42^\circ\ 16'$$
$$e = 0.016$$
$$\log \mu = 9.89566 \text{ or,}$$
$$\mu = 45^\circ.059$$
$$T = 1901, \text{ December } 1.821$$
$$a \sin i = 15{,}901{,}000 \text{ km.}$$

Owing to the very low value of the eccentricity, ω and T are the most uncertain of these elements, but even in these cases the errors should not be large.

An ephemeris was computed with these elements, and the differences between the observed and the computed velocities are given in the column O. – C. of the table above. In view of the character of the spectrum measured, these residuals are entirely satisfactory. A least squares solution might reduce them slightly, but as no significance is to be attached to fractions of a kilometer in measures of stars having this type of spectrum, it has not seemed desirable to undertake it. The accompanying diagram shows the curve derived from the set of elements, and the positions of the observed velocities in reference to it. Owing to the high velocities involved, the scale is necessarily small, but the general agreement of the values is well shown.

In conclusion attention may be called to the fact that *η Orionis* is also a visual double star with components of the fourth and sixth magnitudes. It is, of course, with the brighter of these that we are dealing here.

YERKES OBSERVATORY,
December 20, 1902.

HERSCHEL'S NEBULOUS REGIONS.[1]

By Isaac Roberts.

William Herschel's observed nebulous regions, fifty-two in number, are in this paper compared with the writer's photographs of the same regions, taken, simultaneously, with the 20-inch reflector and the 5-inch Cooke lens.

The nebulous regions referred to in the above title were described by William Herschel in a paper which he communicated to the Royal Society in the year 1811, and which was published in the *Philosophical Transactions*, Vol. **74**, under the heading "Construction of the Heavens."

So far as I can gather, no systematic efforts were made to verify Herschel's observations of these fifty-two regions until six years ago, when the work of photographing them was commenced at my Observatory, using for the purpose the 20-inch reflector and the 5-inch Cooke lens.

The photographs were taken in duplicate, simultaneously, with exposures of ninety minutes' duration, and at times when the objects were as near as practicable to the meridian and the sky clear during the exposure. The plates were selected and tested for sensitiveness, so that with the reflector the images of stars to about the seventeenth magnitude would appear on the plates and stars of about the fifteenth magnitude would appear on the plates exposed with the Cooke lens.

My long previous experience in photographing the heavens enabled me to judge that under these conditions nebulosity of at least the degree of faintness that could be seen by Herschel with his two- and four-foot reflectors would be shown on the photographic plates.

The tabular method adopted by Herschel in publishing the results of his telescopic observations enables me to give the photographic results in a concise and intelligible form, coinciding line by line with his, by comparing the headings and reading the descriptive matter relating to each object respectively.

[1] The manuscript of this article was accepted by the editors on the supposition that it was not to be published in other current journals.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.

Herschel's Nos.	R. A. (1900)			Dec. (1900)		Herschel's description in *Phil. Trans.*, 1811	Dates when photographs were taken	Isaac Roberts' descriptions of his photographs
1	0^h	10^m	8^s	9°	26′	Much affected with nebulosity	1900, Nov. 22	Sky clear; stars small and faint, and few in number; large areas void of stars; no nebulosity on plate.
2	0	17	37	3	59	Much affected	1899, Sept. 5	Sky clear; stars small and faint, and not very numerous; large areas void of stars; no nebulosity on plate; film dark.
3	0	22	23	29	9	Affected	1899, Sept. 9	Sky clear; stars small and very numerous; one star of 5.9 mag., *B. D.* 28°.75, on plate; small areas void of stars; no nebulosity.
4	0	25	37	3	59	Much affected	1900, Nov. 22	Sky clear; stars few and faint; large areas void of stars; no nebulosity on plate.
5	0	30	11	23	25	Much affected	1900, Oct. 27	Sky clear; stars faint and numerous; nebulæ H *III*, 476, and *N. G. C.* 169, d'Arrest and Ld. R., together with other fainter ones on plate; many areas void of stars; no diffused nebulosity.
6	0	36	28	0	29	Appeared to be affected with very faint nebulosity	1899, Oct. 28	Sky very clear; stars small and very few in number; large areas void of stars; some small nebulæ on plate; no diffused nebulosity.
7	0	38	0	41	10	Affected with nebulosity	1895, Oct. 17	Sky very clear; stars crowded on plate; many small areas void of stars; several photographs have been taken of this region, which includes the great *Andromeda* nebula *M.* 31, part of the *n. f.* end of which would cross Herschel's field of view in this sweep.
8	0	39	27	39	16	Unequally affected	1900, Oct. 17	Sky clear; stars crowded on plate; many small areas void of stars; part of *s. p.* end of *M.* 31 on plate; no other diffused nebulosity.
9	0	41	19	43	30	Suspected faint nebulosity	1900, Oct. 26	Sky clear; stars small and crowded on plate; many small areas void of stars; no diffused nebulosity.
10	0	48	38	43	35	Suspected faint nebulosity	1900, Oct. 26	Sky clear; stars small and crowded on plate; numerous areas void of stars; nebula *N. G. C.* 317 on plate; no diffused nebulosity.
11	1	41	8	29	48	Suspected to be tinged with milky nebulosity	1900, Nov. 27	Sky clear; stars small and numerous; large areas void of stars; no nebulosity.
12	2	27	55	19	0	Much affected with nebulosity	1900, Dec. 13	Sky clear; stars small and not very numerous; large areas void of stars; some very small and faint nebulæ on plate; no diffused nebulosity.
13	4	2	14	25	11	Much affected	1901, Feb. 13	Sky very clear; stars small and numerous; large areas void of stars; no nebulosity.
14	4	23	51	35	7	Suspected pretty strong nebulosity	1901, Feb. 13	Sky very clear; stars small and crowded on *s.* and *s. p.* sides, but few on the rest of the plate; large areas void of stars; nebula H *I* 217 and also a tenth mag. star surrounded by very faint nebulosity 11′.5 *n. f.* H *I* 217 on plate; no nebulous region.
15	4	24	51	35	8	Suspected nebulosity		
16	4	26	29	−7	30	Strong milky nebulosity	1901, Feb. 14	Sky clear; stars small and very few on plate; large areas void of stars; no nebulosity.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.—*Continued.*

Herschel's Nos.	R. A. (1900)			Dec. (1900)		Herschel's description in *Phil. Trans.*, 1811	Dates when photographs were taken	Isaac Roberts' descriptions of his photographs
17	4	29	2	20	50	Much affected	1901, Feb. 15	Sky very clear; stars small and very numerous; small areas void of stars; no nebulosity.
18	4	44	5	20	50	Much affected	1901, Feb. 15	Sky very clear; stars small and crowded on plate; small areas void of stars; no nebulosity.
19	4	52	17	26	45	Strong suspicion of very faint milky nebulosity	1901, March 9	Sky clear; stars small and very few; large areas void of stars; no nebulosity.
20	5	15	50	25	1	Very much affected	1901, M'ch 12	Sky clear; stars small and very few on plate; large areas void of stars; no nebulosity.
21	5	19	20	25	1	Affected		
22	5	28	53	-6	56	Affected with milky nebulosity	1901, M'ch 13	Sky clear; stars not very numerous; large areas void of stars; H *IV* 33 *Orionis* on plate; no nebulosity.
23	5	30	10	-2	43	Affected	1901, M'ch 12	Sky clear; stars small and very few; large areas void of stars; no nebulosity.
24	5	31	56	-4	18	Visible and unequally bright nebulosity. I am pretty sure that this joins to the great nebula in *Orion*	1902, M'ch 5	Sky very clear; stars small and not very numerous; areas void of stars; no nebulosity on plate.
25	5	35	34	-2	31	Diffused milky nebulosity	1900, Jan. 25	Sky clear; stars very numerous on *p.* half of plate, but few on *f.* half, where there are large areas void of stars; large cloud of nebulosity *n. f.* ζ *Orionis* with broad division void of stars, but with some nebulosity in *s. f.* to *n. p.* direction; other divisions break up the cloud into separate masses. To the *s.* of ζ is a stream of nebulosity, 54 minutes of arc in length, with an embayment free from nebulosity dividing it in halves. Another faint nebulosity extends from ζ 27 minutes of arc toward the *s.*, *s. p.* and *n. p.* The star *B. D.* — 1°1001 is in the midst of nebulosity, and it has a companion on the *s. p.* side. The star *B. D.* — 1°1005 is involved in a large cloud of streaky nebulosity, and it has a companion on the *p.* side. The star *B. D.* — 2°1345 is H *IV* 24, *N. G. C.* 2023; it is in the midst of a large, dense streaky cloud of nebulosity which has in it condensations and remarkable rifts free from nebulosity; near the *s.* end of one of these rifts is a twelfth magnitude star. The star *B. D.* — 2°1350 is in the midst of a cloud of nebulosity with some faint structure in it, and it has a companion on the *n. p.* side. The region here referred to, which covers four square degrees of the sky, has so many remarkable features that it is necessary, in order to make it intelligible to the reader, to present the photograph annexed along with the above description.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.—*Continued.*

Herschel's Nos.	R. A. (1960)			Dec. (1900)		Herschel's description in *Phil. Trans.*, 1811	Dates when photographs were taken	Isaac Roberts' descriptions of his photographs
26	5^h	36^m	52^s	-6°	$57'$	A pretty strong suspicion of nebulosity	1901, Mar. 22	Sky clear; stars small and few; large areas void of stars; no nebulosity.
27	5	43	11	+1	8	Affected with milky nebulosity	1901, Mar. 13	Sky clear; stars very few in number; large areas void of stars; no nebulosity.
28	6	1	1	+3	44	Much affected.	1902, Jan. 29	Sky clear; stars crowded on *n. p.* and *s. p.* sides; large areas void of stars; no nebulosity.
29	6	0	54	−20	27	Affected	1902, Mar. 6	Sky clear; stars small and very numerous; many areas void of stars; no nebulosity.
30	6	40	7	41	16	Affected	1901, Mar. 22	Sky clear; stars few in number; large areas void of stars; cluster H. *VIII*, 71 on plate; no nebulosity.
31	9	27	32	−18	27	Affected	1902, Mar. 6	Sky clear; stars small and few in number; large areas void of stars; no nebulosity
32	9	36	43	71	13	Much affected with very faint whitish nebulosity	1901, Aprl. 12	Sky clear; stars small and numerous; several large areas void of stars: no nebulosity.
33	10	11	50	−9	3	Very faint whitish nebulosity	1901, Aprl. 15	Sky clear; stars small and numerous; large areas void of stars; no nebulosity.
34	10	22	25	51	32	Much affected	1901, Aprl. 13	Sky clear; stars small and not numerous; large areas void of stars; no nebulosity.
35	10	40	59	62	45	Affected with very faint nebulosity	1901, Aprl. 14	Sky clear; stars small and not very numerous; large areas void of stars; no nebulosity.
36	11	4	30	62	44	Affected	1901, Aprl. 15	Sky clear; stars small and numerous; areas void of stars; several small, faint nebulæ on plate; no diffused nebulosity.
37	12	2	5	30	37	Affected with whitish nebulosity	1901, Aprl. 17	Sky clear; stars small and few in number; large areas void of stars; H.*II*, 321, and H.*II*, 802, on plate; no nebulosity.
38	12	12	40	30	37	Affected with whitish nebulosity	1901, Aprl. 18	Sky clear; stars few in number; large areas void of stars; four small prominent nebulæ on plate; no diffused nebulosity.
39	13	12	15	34	8	Much affected	1901, Aprl. 17	Sky clear; stars not very numerous; large areas void of stars; no nebulosity.
40	14	2	20	34		Very much affected and many faint nebulæ suspected	1899, June 2	Sky clear; stars small and not numerous; areas void of stars; no nebulosity.
41	15	9	37	18	57	Affected with very faint nebulosity	1899, June 12	Sky clear; stars small and not very numerous; areas void of stars; no nebulosity.
42	21	3	26	−1	53	Much affected with whitish nebulosity	1902, Nov. 4	Sky clear; stars very numerous; no nebulosity.
								Herschel's sweep 42, as given in the *Phil. Trans.* (*R. A.* (1800), 20^h 58^m 20^s *N. P. D.* (1800) = 92° 17′) is not in sequence; as this may be due to a typographical error in one of the co-ordinates, a plate corresponding to *R. A.* (1800) = 20^h 38^m 20^s, *N. P. D.* (1800) = 92° 17′ was taken on Aug. 28, 1897, as follows:
							1897, Aug. 28	Sky very clear; stars crowded on plate; no nebulosity.
43	20	53	15	16	44	A good deal affected	1897. Oct. 20	Sky clear; stars crowded on plate; no nebulosity.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.—*Continued.*

Herschel's Nos.	R. A. (1900)			Dec. (1900)		Herschel's description in *Phil. Trans.* 1811	Dates when photographs were taken.	Isaac Roberts' descriptions of his photographs
44	20	54	34	43	32	Faint milky nebulosity scattered over this space, in some places pretty bright	1896, Oct. 10	Sky very clear; stars crowded on parts of plate; large areas void of stars on others; Nebula, H. *V* 37, *N.G.C.* 7000 forms part of this region; the photograph shows it as a magnificent object. I have published a photograph of this region in Vol. II of *Stars, Star-Clusters* and *Nebulæ*, pl. 24, p. 155, and also in *Knowledge*, Nov. 1, 1898; a copy is also annexed to this paper.
45	20	57	34	−1	34	Much aeffcted with whitish nebulosity	1897, Sept. 21	Sky clear; stars small and numerous; no nebulosity.
46	20	56	55	43	16	Suspected nebulosity joining to plainly visible diffused nebulosity	1896, Oct. 10	Regions 44 and 46 are on the same plate; see description given above, No. 44.
47	21	5	8	14	21	Affected	1899, Aug. 6	Sky clear; stars small and crowded on plate; no nebulosity.
48	21	34	15	10	19	Much affected	1898, Oct. 12	Sky clear; stars small and numerous; areas void of stars; no nebulosity.
49	21	46	52	21	31	Affected	1899, Aug. 9	Sky clear; stars small and crowded; areas void of stars; no nebulosity.
50	22	57	24	25	45	Much affected	1898, Sept. 20	Sky clear; stars very numerous; areas void of stars; no nebulosity.
51	22	57	54	25	45	Affected		
52	23	0	17	29	17	A little affected	1902, Oct. 27	Sky clear; stars small and very numerous; areas void of stars; H. *II*. 212, on plate; no diffused nebulosity.

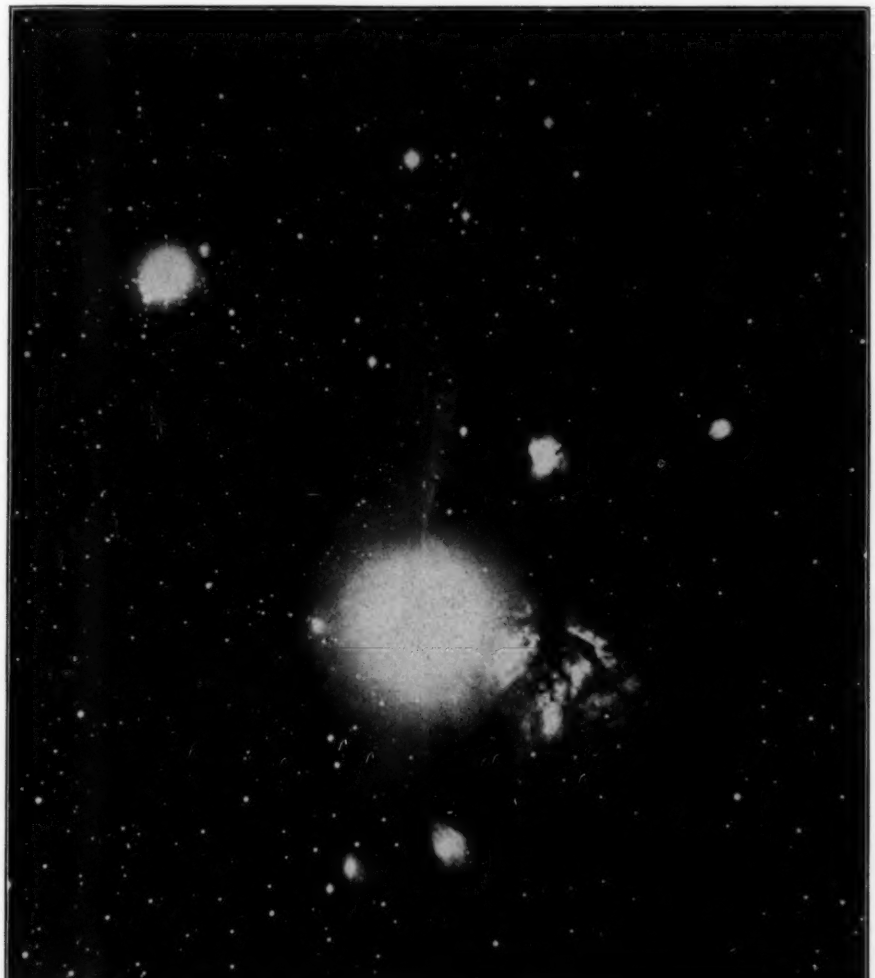
CONCLUSION.

The final results of the correlation of Herschel's nebulous regions and my photographs can be given in a few words as follows:

Of the fifty-two nebulous regions described by Herschel, the photographs show diffused nebulosity on four of them only; there is no visible trace of diffused nebulosity on forty-eight of the areas, but on the remaining four, which are Nos. 7, 25, 44, and 46, respectively, in the table, there is nebulosity with remarkable characteristic features, and these are delineated upon three of the photographs, regions Nos. 44 and 46 being on one plate.

STARFIELD, CROWBOROUGH, SUSSEX,
November 1902.

PLATE IV.



NEBULÆ AROUND ζ *ORIONIS* (January 25, 1900).

PLATE V.

NEBULA HERSCHEL V 37 *CYGNI* (October 10, 1896).

DIFFUSED NEBULOSITIES IN THE HEAVENS.

By E. E. Barnard.

In the present number of this Journal there is a paper by Dr. Isaac Roberts containing an account of a photographic investigation of certain regions of the sky supposed by Herschel to be affected with diffused nebulosity. This investigation was made simultaneously with Dr. Roberts' 20-inch (51 cm) reflector and 5-inch (12.7 cm) Cooke portrait lens. The exposures given by Dr. Roberts to test the existence of these diffused nebulosities was ninety minutes, which, it is stated, showed stars of the sixteenth and seventeenth magnitudes. Out of these fifty-two regions he found only four that showed any traces of nebulosity. These four regions, however, had already been shown by numerous photographs to be nebulous. One of these, the great nebula north of *α Cygni*, was first photographed by Dr. Max Wolf some twelve years ago and has lately been called by him the "America Nebula"[1] from its striking resemblance to North America as shown on maps and globes.

The curious nebulous ribbon extending southward from ζ *Orionis* seems to have been first photographed by Professor W. H. Pickering and others as far back as 1889.

This question of large areas of diffused nebulosity in the sky is a very important one, not yet fully appreciated, but which must sooner or later have the highest bearing on a proper understanding of the physical condition of the universe. Dr. Roberts' negative results are so sweeping in character that it is highly important that anything tending to prove the existence of any of these questioned regions of nebulosity should be brought forward at once.

First, I do not think 90 minutes a sufficient exposure to test the existence of some of these nebulosities with Dr. Roberts'

[1] The "North America Nebula" would perhaps be more definite, for it is North America to which Dr. Max Wolf intends the compliment.

outfit. The photographing of sixteenth or seventeenth magnitude stars does not necessarily prove that the same exposure ought to show diffused nebulosities, for photographing faint stars often stands on an entirely different footing from photographing faint nebulosities.

Second, it is a little unreasonable to suppose that Herschel, who made so few blunders compared with the wonderful and varied work that he accomplished, should be so palpably mistaken in forty-eight out of fifty-two observations of this kind.

I have myself been very much interested in the diffused nebulosities of the sky and have independently come across some of these very regions of Herschel, besides others not noted by him. It has been a long-cherished desire of mine to investigate them further photographically, and I now hope to be able to put this desire into a practical reality within the next twelve months. Some of these regions I have already shown to be extraordinary features of the sky—as instances, the nebulous regions of 15 *Monocerotis*, of ρ *Ophiuchi*, the region surrounding the *Pleiades*, etc. All of these were known to me previous to their really being proved by the photographic plate and the portrait lens to be true nebulosities.

As far back as January 1892, in *Knowledge*, **15**, 14–16, I called special attention to these nebulous regions of Herschel and gave the table contained in Dr. Roberts' present article. Attention was called to these objects as being suitable for photographic investigation in these words:

> It would appear that this table of diffused nebulosities will just now be of extremely great value, as it at once points out to those interested in photographing such objects, the proper pointings of their exposures. I have taken the liberty to copy the foregoing table in full for the benefit of those not familiar with it and who may wish to try exposures on these objects.

What leads me to hope that more of these regions given by Herschel may yet be shown to be nebulous with photographic plates, is that one of these very objects, which the photographs of Dr. Roberts show to be free from nebulosity, is really the brightest portion of one of the most extraordinary nebulæ in the sky, as shown by photographs made by two independent observers with

three different photographic telescopes on several different occasions. I refer to region 27 of the list. This is described by Herschel as being "affected with milky nebulosity" and its position given for 1800.0 as α $5^h\ 38^m\ 5^s$; P. D. 88° 55′.

The right ascension and declination for 1900.0 would be closely

$$\alpha = 5^h\ 43^m\ 13^s;\ \delta = +1°\ 8'.$$

Dr. Roberts' note on this is: "Sky clear; stars very few in number; large areas void of stars; no nebulosity."

In *Popular Astronomy*, **2**, 151–154, December 1894, the writer has given some experiments with a very small magic lantern lens in photographing diffused nebulosities. In this an account is given of the finding of a great nebula extending in a curved form over the entire body of *Orion*. The brightest portion of the nebula is near 56 and 60 *Orionis*. From the photographs, the position of this brightest portion is in

$$1900.0\ \ \alpha = 5^h\ 43.7^m;\ \delta = +1°\ 0'.$$

This would make it identical with Herschel's No. 27. These pictures were made with an ordinary child's magic lantern lens, of 1.5 inches diameter and 4.9 inches equivalent focus. The exposures were 1894, October 3, for $2^h\ 0^m$, and October 24, for $1^h\ 15^m$. The shortest exposure, $1^h\ 15^m$, showed it best. I suppose a half-hour's exposure would have shown traces of it. Unknown to me at the time, this nebula had already been photographed in 1889 by Professor W. H. Pickering on Mount Wilson (altitude 6,250 feet) in southern California, with a Voigtlander portrait lens of 2.6 inches aperture and 8.6 inches equivalent focus, with an exposure of three hours.[1]

Besides the photographs made with the magic-lantern lens, I have two made with the six-inch Willard portrait lens that show portions of the nebula distinctly. They were made 1893, October 17, with $3^h\ 0^m$ exposure; and 1894, October 3, with $2^h\ 0^m$ exposure. In both the photographs with the Willard lens, the region of the nebulosity falls near the edge of the plate and hence the stars are deformed. At the time of making these

[1] See *Sidereal Messenger*, **9**, 1, 1889.

pictures I knew nothing of the existence of this object and therefore had no choice as to its location on the plate. One of these photographs has been selected for reproduction, though it will necessarily be unsatisfactory because of the position of the object close to the edge of the plate.

[Through the courtesy of the editor of *Popular Astronomy*, the map of *Orion* showing the location of the nebula is here reproduced. (Plate VI, A).]

To further show the reality of the nebulous region No. 27 of Herschel's list, I made an exposure of $2^h\ 10^m$ on the night of January 17, 1903, on the region with a small, cheap lantern lens belonging to Professor Hale. This lens is 1.6 inches in diameter and has an equivalent focus of 6.3 inches or $\frac{a}{f}=\frac{1}{4}$.

The sky was clear and the conditions fair. The resulting negative showed a fairly good field of nearly $25°$.

Most of the great curved nebula is clearly shown, especially the region described by Herschel, which is now in question and which, as I have said, is the brightest portion of the nebula.

This plate does not show the nebula as strongly as those made with the lantern lens used by me at the Lick Observatory, because, for one reason, the present lens is relatively of smaller angular aperture, $\frac{1}{4}$ as compared with $\frac{1}{3.3}$; and for another reason the sky was not perhaps as pure; but the main portion of the nebula is conspicuous. There is therefore no question but that this nebulosity exists where Herschel saw it.

Just what the faintest stars are that appear on this plate I am not prepared to say, not having had any chance to make a comparison with the sky; but they are certainly several magnitudes brighter than the fifteenth or sixteenth magnitude.

It was with the same instruments described in his present paper that Dr. Roberts failed to get any traces of the exterior nebulosities of the *Pleiades*, which have been shown by four observers with four different instruments not only to exist, but to be not at all difficult objects.

YERKES OBSERVATORY,
January 1903.

PLATE VI.

N.

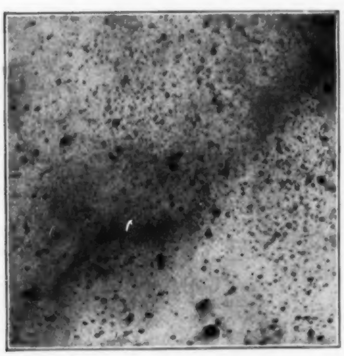
E.

W.

A.

N.

E.



W.

B.

THE GREAT CURVED NEBULA IN ORION.

A.—Drawing from two lantern lens photographs (Oct. 3 and 24, 1894).

B.—Photograph, with Willard lens, of region inclosed in the square in the drawing above (Oct. 17, 1893).

MINOR CONTRIBUTIONS AND NOTES.

NEW COLLABORATORS OF THE ASTROPHYSICAL JOURNAL.

THE ASTROPHYSICAL JOURNAL has from the outset enjoyed many advantages from the cordial co-operation of eminent investigators at home and abroad. The list of collaborators includes many of the ablest representatives of the branches of science with which the JOURNAL is most directly concerned. On the death of M. Cornu, who had assisted the JOURNAL in many ways, Professor Bélopolsky kindly accepted an invitation to fill the vacant place. As the editors have always desired to maintain a fairly even balance between the physical and the astronomical sides of the subject, they now take special pleasure in announcing that Professor Kayser and Professor Runge have signified their willingness to serve as collaborators. With this added strength on the laboratory side, the JOURNAL will continue to emphasize the importance of treating celestial and terrestrial problems of radiation from a single point of view.

A CONTRIBUTION TO THE KNOWLEDGE OF SPECTROSCOPIC METHODS.[1]

THE investigation undertaken by Dr. Konen proposes to test the method of the electric discharge under liquids as a means of solving the important problem of the origin of band-spectra. His paper touches upon the difficulties which presented themselves to former investigators — notably the transitory nature of band-spectra, their faintness, and the difficulty of ascertaining just what element or elements were essential to the production of any given spectrum; it mentions the methods and apparatus previously employed — namely, Geissler tubes and a flame or a carbon arc in closed receptacles; and states that we should expect this method of discharge under liquids to present fewer difficulties owing to the presence of fewer impurities.

The discussion of the *arc* discharge is preceded by a brief résumé

[1]Abstract, by Norton A. Kent, from advance proofs (furnished by the author) of a paper to appear in *Annalen der Physik*.

of the work of Davy, Masson, and Liveing and Dewar; and attention is called to the fact that very few investigations other than visual have been made upon this subject. The apparatus used by Konen consisted of a glass vessel silvered within, blackened without, and fitted with a quartz window. The pole pieces were held in brass clamps. The current employed varied from 15 to 20 amperes (below 15 the arc ran poorly, while above 20 the liquid was unnecessarily heated). The light passed through a quartz window and was focused by a quartz lens upon the slit of (1) a Steinheil prism (flint glass) spectroscope, in the first part of the investigation, and (2) later and in most of his work, a Rowland concave grating of one meter radius of curvature, rigidly mounted and used in the first spectrum — the extent of region covered being from $\lambda 5500$ to $\lambda 1800$. The spectrum was photographed on films.

As a result of the tearing asunder of the electrodes, the immersing water is rendered turbid, especially so where carbon or iron poles are used; all liquids are befouled sooner or later; water, alcohol, and electrolytes last the longest; some liquids admit of filtration; all clear by standing.

Besides a continuous spectrum there are always present (1) the H and K lines of calcium, also generally other lines of the calcium spectrum; (2) the two chief aluminum lines; and (3) the D lines of sodium. With metal poles *Fe*, *Cu*, *Ba*, *Na*, *K*, *Li*, *Tl*, and brass, the characteristic line spectra appear — but band spectra never. The liquid-arc lines are generally fully as sharp as those of the air-arc; many are sharper. The relative intensity varies and some air-arc lines are wholly absent in the liquid-arc spectrum. No displacement is apparent. The nature of the surrounding liquid has no influence upon the spectrum given by metal poles — this being true for alcohol, distilled water, and glycerine — and, with certain reservations, for salt solutions. In concentrated $BaCl_2$ and $CaCl_2$ solutions the strongest barium and calcium lines are present, but they are incomparably weaker than in air, and only the strongest calcium lines are reversed. The effect is not a function of decrease of temperature, nor of the behavior of the poles, *i. e.*, whether merely a single point or the whole pole glows. The probable physical and chemical processes which take place when carbon poles are used in a $NaCl$ solution is as follows: Near the poles the liquid is in the spheroidal shape; the $NaCl$ remains in solution; the glowing carbon and the water vapor react to produce CO and H, etc.; and these gases form the medium in which the true arc burns.

Vaporized liquid surrounds these gases, being forced aside by the latter, which are better conductors by reason of their hotter and ionized condition. Only the gases of the poles enter. This is further shown by the use of cored carbons filled with salt — an arrangement which gives the sodium spectrum.

The problem of the origin of the so-called carbon and cyanogen bands is then attacked. The carbon arc in air gives bands whose edges lie at λ 4606.33, 4216.12, 3883.55, 3590.48. These are generally ascribed to cyanogen. The band at λ 3883 photographs most quickly, and is therefore used as a test of the presence of the cyanogen spectrum. By using boiled water and boiled electrodes Konen succeeded in causing all four of these bands to disappear. Small amounts of air suffice to produce weak traces of them; therefore they will appear if unboiled carbons or conducting water be used, or if an excess of air be blown into the liquid, or, further, if the arc be submerged in strong water solutions of ammonia. In alcohol and CCl_4 the "cyanogen" bands are lacking; in petroleum a trace of λ 3883 appears when the poles used have been made from carbon rods glowing under aniline. In KNO_3 only traces appear — even weaker than those given by water containing air. But in view of what has been said of the part played by salt solutions in the development of the arc, this negative result is not surprising. *In every case the presence of nitrogen* is necessary to the production of the bands in question. This forms a new proof that these four bands belong to cyanogen.

As to the origin of the Swan spectrum — a much-contested question for over fifty years — Stokes advanced the hypothesis that the so-called "carbon" band spectrum belonged to carbonic oxide — an hypothesis the truth of which Konen considers not yet rigorously proven.

To facilitate the discussion of this question six classes of spectra are distinguished :

1. A continuous spectrum without bands or lines.
2. A line spectrum.
3. The cyanogen spectrum with edges at λ 3590, 3883, 4216, and 4606.
4. The so-called "carbon" band, or Swan spectrum, with principal heads at λ 4382, 4738, 5164, 5633, and 6187.
5. The so-called "carbonic oxide" spectrum with edges at λ 4509, 4834, 5196, 5608, 6078, 6399, 6622.
6. The carburetted hydrogen flame spectrum with edges λ 4315, 4368, 3872, and 3627.

The condition under which the first three appear are briefly given. No. 4 appears with No. 6 in the carburetted hydrogen flame; in the arc in air and different gases; in the liquid arc under certain conditions; in vacuum tubes filled with carburetted hydrogen; in the cyanogen flame; in carbon-dioxide gas at moderate pressures; in tubes during the first part of the discharge in carbonic-oxide gas; in the brush discharge in alcohol, glycerine, ether, acetic acid, etc.; in the Sun; and, doubtfully, in numerous other instances. The Swan spectrum is absent in the CO and CS_2 flames. No. 5 is obtained from an impurity in Geissler tubes; in CO and CO_2 tubes; also with oxygen or air in various carbon compounds. It is absent in the CO and CS_2 flame, and in the spark of hydrogen or water vapor. No. 6 is obtained only by burning carburetted hydrogen with oxygen in, *e. g.*, a Bunsen flame. It is absent in the spark discharge between carbon electrodes in air, CO_2 and H; also when No. 4 is strongly present. It is a remarkable fact that most observers have identified the flame or Swan spectrum with No. 4.

Spectrum No. 4 is then discussed. The question of its origin is connected closely with that of No. 5. The points to be settled are: Does the spectrum belong to (1) carbon itself; (2) to the positive ion of carbon in an ionized state; (3) to a hydrogen compound of carbon; or (4) to an oxygen compound—*i. e.*, to CO? (1) and (2) may be treated together. The results obtained with cyanogen and water, etc., show that (3) is out of the question. Moreover as no rule has been found whereby a certain spectrum may be shown to be due to one of two components, or to the "completed compound," or to all these together, it is clear that some uncertainty must enter into the proof that a definite compound causes that spectrum. In the light of this reasoning Smithell's investigation, in which he sought to explain the behavior of CO and CO_2 by different degrees of dissociation, is very artificial and, at best, hypothetical. If it cannot be shown indubitably that the spectrum in question comes from a finished product, then the proof that it is caused by the presence of CO, for instance, means no more than that one or the other element is essential.

The real question is: "Is the presence of oxygen necessary to the formation of the 'carbon' band-spectrum?" Two experimental difficulties are present. An affirmative result—the presence of a certain band in a given case—admits of no conclusion; while the value of a negative result—the absence of that band—depends wholly upon the sensitiveness of the spectral reactions. If this limiting sensitiveness be passed it is practically and chemically impossible to exclude the

presence of a definite substance. But it is possible to establish one limit showing how much of an impurity must be present to produce a given spectrum, and another showing how much can be present without being active spectroscopically. Only within such bounds has the following experiment any meaning; and it *cannot presume to lead to a decision in the question of the origin of spectra 4 and 5.*

Metal and carbon poles were used. The latter were placed in a glass tube attached to a mercury pump; the air was exhausted, the tube steadily heated, and an electric discharge — as strong as possible — was passed between the pole pieces. Then the tube was sealed off and broken open beneath the surface of a testing liquid; the terminals were then placed in position, boiled in liquid, and covered with a layer of graphite by the action of the arc discharge — a result most easily accomplished by the use of aniline. The liquids used — water, salt solutions, alcohol, glycerine, carbon tetrachloride, aniline, carbon bisulphide, petroleum, and ammonia — were chemically pure and, as far as possible, were preserved over phosphorous pentoxide in the dark. In distilled and conducting water, spectrum No. 4 appeared complete. As was suspected, CO is developed in large amounts. An air-blast made no difference. The same is true for alcohol, glycerine, and ammonia. The bands are less intense than in air. Metal terminals cause a further weakening. In some liquids, *e. g.*, very strong salt solutions, the bands can be blotted out. It is well known that in air the introduction of metals weakens the carbon and cyanogen bands. In both cases it is quite appropriate to explain the phenomenon in accordance with the CO hypothesis, as a process of oxidation in which the metal seizes the oxygen. The temperature of the carbon poles under water plays a part. With metals this temperature is visibly less, as is clear from the smaller consumption of carbon and the shrinking of the heated area of the pole, and it is in precisely these instances that the carbon bands are lacking. Carbon bisulphide fogs quickly, permitting only visual observations. In all cases the carbon bands are unquestionably present. In aniline bands appear at λ 56, 51, and 47, but the decomposition produces such strong absorption near λ 3883 that that region of the plate is not affected at all, and the cyanogen band therefore does not appear. In carbon tetrachloride these three bands are stronger than in aniline; they are most intense in petroleum and here a trace of λ 3883 appears, while the bands of spectrum No. 6 as well as all lines are absent. Visual observation of carbon bisulphide, chloroform, and turpentine give the same result, apart from the continuous spectrum the green band alone is present, and it only faintly.

If the sensitiveness of the arc to traces of water or oxygen is small enough, then the deduction may be drawn that "the presence of oxygen is not necessary for the production of spectrum No. 4. The old theory is then confirmed — *the so-called carbon band-spectrum rightly deserves its name.*"

On the other hand, we must emphasize the impossibility of keeping the carbon rods and the liquids free from oxygen; and, if we rate higher the sensitiveness of the liquid arc, we must abstain from any conclusion. If the existence of impurities is assumed, an exceptional position must be conceded to oxygen and carbonic acid.

Dr. Konen discusses other forms of discharge in liquids, namely, the *glow*, *brush*, and *uncondensed* and *condensed sparks*. The points of greatest interest in connection with the subject in hand are that with the *brush* discharge in ether, alcohol, and solutions of iodine or bromine in ether, the Swan spectrum is obtained in an especially beautiful manner. In the arc the electrodes were the important element, in the brush discharge the liquid is. "The only deduction possible 'from brush discharge experiments' must take the form of a reiteration of the statement made regarding the results of the carbon arc."

With the *condensed spark* it is impossible with water, ether, alcohol, and salt solutions, and either carbon or graphite electrodes, to obtain traces of the carbon or cyanogen spectrum; and here, as with the arc, salt solutions have no effect. With a 9.5 per cent. $BaCl_2$ solution and Fe terminals Konen obtained a bright line spectrum where Hale had found an absorption spectrum.

H. KONEN.

FIRE AT THE YERKES OBSERVATORY.

DURING the past two years a horizontal reflecting telescope, to be used with a cœlostat of 30 inches aperture, has been in process of construction at the Yerkes Observatory. In conjunction with the instrument a large concave grating spectrograph had been provided for photographing stellar spectra. On December 22, when the final adjustments of the spectrograph were being made, the building caught fire from the breaking down of the insulation of the wires connected with the comparison spark apparatus. Under ordinary circumstances the small blaze thus produced might easily have been extinguished. But it had been necessary to construct the constant-temperature spectrograph house and the building containing the telescope as cheaply as

possible as very little money could be obtained for the purpose. For this reason building paper and wood, with intervening air spaces, had been employed. This burned so rapidly that although the fire extinguishers were at once brought into action the flames were past control. In spite of the best efforts of the members of the Observatory staff, the buildings were destroyed, together with the new spectrograph and the 30-inch plane mirror, driving-clock, and all the smaller parts of the cœlostat. A second plane mirror of 24 inches aperture was saved, but the 24-inch concave mirror of 62 feet focal length was destroyed.

The house containing the horizontal telescope was situated at some distance from the main Observatory building, which was not damaged in the least.

The work of reconstructing the mechanical and optical parts is already under way, and it is hoped that funds can be secured to complete the instrument in a thoroughly satisfactory manner, and to provide a suitable house of permanent construction.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to uniformly employ the metric units of length and mass; the English equivalents may be added if desired.

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